Integration Gain of Heterogeneous WiFi/WiMAX Networks

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Abstract

We study the integrated WiFi/WiMAX networks where users are equipped with dual-radio interfaces that can connect to either a WiFi or a WiMAX network. Previous research on integrated heterogeneous networks (e.g., WiFi/cellular) usually consider one network as the main, and the other as the auxiliary. The performance of the integrated network is compared with the "main" network. The gain is apparently due to the additional resources from the auxiliary network. In this study, we are interested in *integration* gain that comes from the better utilization of the resource rather than the increase of the resource. The heterogeneity of the two networks is the fundamental reason for the integration gain. To quantify it, we design a generic framework that supports different performance objectives. We focus on the max-min throughput fairness in this work, and also briefly cover the proportional fairness metric. We first prove that it is NP-hard to achieve integral max-min throughput fairness, then propose a heuristic algorithm, which provides 2-approximation to the optimal fractional solution. Simulation results demonstrate significant integration gain from three sources, namely spatial multiplexing, multi-network diversity, and multi-user diversity. For the proportional fairness metric, we derive the formulation and propose a heuristic algorithm which shows satisfactory performance when compared with the optimal solution.

Index Terms

WiFi, WiMAX, Heterogeneous network, Integration gain, NP-hardness, Approximation algorithm.

I. Introduction

The IEEE 802.16 (WiMAX) is a promising technology due to its high data rate, wide coverage, and built-in support for mobility and security. Given the current vast deployment of WiFi

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networks, the coexistence between WiFi and WiMAX is inevitable. Major companies such as Intel and Motorola are promoting the integrated WiFi/WiMAX interface to take advantage of such scenario [10]. Users equipped with such interfaces can associate with a WiMAX base station (BS) or a WiFi access point (AP). Compared to the scenario where users only connect to WiFi networks, the benefit of the integrated network is obvious: we have additional spectrum resource from the WiMAX network. However, a closer look suggests that we may be able to reap significant gain from the heterogeneity of these two networks in addition to the extra resource. For example, in a typical integrated WiFi/WiMAX network, a WiMAX BS may cover a service area with up to hundreds of WiFi APs in it. In the WiFi network, users may experience poor quality of service (QoS) in some congested APs, while in some other APs, capacity may not be fully utilized. Similarly, in the WiMAX network, per-user throughput could be low if the number of WiMAX users is large. If users have the flexibility to switch between WiFi and WiMAX networks using the integrated interface, some WiFi users can switch from congested APs to WiMAX, while some WiMAX users can switch to under-utilized WiFi APs. Thus, the QoS in both networks improve. We refer to this as the *spatial multiplexing gain*. In addition, a user may have a low WiFi rate and a high WiMAX rate or vice versa. If the user intelligently selects its association, the network capacity improves, which is referred to as *multi-network diversity gain*. Furthermore, multiple users could be switched from their current associated network to the other one to improve the overall network performance. There exists an order to switch these users following which the gain can be maximized. We refer to it as the *multi-user diversity gain*. These three types of improvements, which will be discussed in more detail later, come from network heterogeneity and better utilization of the resource rather than the increase of the resource. This observation motivates our work. Our objective is to identify such an integration gain, which was not addressed in existing work on integrated heterogeneous network (e.g., WiFi/cellular).

Our contributions are as follows. First, we propose a generic framework to identify the integration gain. The framework can serve different objectives. In this study, we focus on the *max-min* throughput fairness, and briefly cover the *proportional* throughput fairness. Second, we prove that it is NP-hard to achieve integral max-min throughput fairness. We propose an approximation algorithm which provides 2-approximation to the optimal fractional solution. The algorithm is shown to achieve significant integration gain, and is easy to implement because the computation and information exchange are distributed. For the proportional fairness metric, we derive the formulation and propose a heuristic algorithm which shows satisfactory performance when compared with the optimal solution.

II. Related Work

In cellular networks, macro/micro cellular architecture is similar to WiFi/WiMAX architecture in terms of spatial heterogeneity. Much work on macro/micro architecture focuses on how to perform optimal handoff [18]. The decision is mainly based on signal strength. The association policies considered in our work can also be viewed as handoff decisions. But instead of signal strength, we make the decision based on network performance metrics, such as throughput fairness. Other work focuses on the resource management and capacity analysis [7], [11]. In general, micro-cells do not bring additional spectrum resources into the original “macro” network. The capacity improvement comes from frequency reuse and intelligent allocation of the resource between macro and micro-cells. In our work, we do not have control over spectrum allocation between WiFi and WiMAX networks. Each network has its own spectrum as well as users. Our objective is to study the integration of the two (separate) networks. From the viewpoint of either one of them, the other does bring additional resource as well as its own users. But we are interested in the integration gain, which is independent of the spectrum resource each network has.

Integrated WiFi/cellular network architecture has also been studied. Usually cellular network has a much smaller bandwidth than that of WiFi network. In most of the work, the cellular network is considered as the main network, and WiFi as the auxiliary. Most research efforts are put on the architecture design and QoS support of such network [17], [13]. Usually, the performance of the integrated WiFi/cellular network is compared with the cellular network where the gain is obvious due to additional resources.

There are a few recent works on the handoff and load balancing in integrated WiFi/WiMAX networks [4], [12]. But none of them explicitly studies the performance gain due to the heterogeneity of the two networks.

AP association in WiFi networks has been extensively studied. The association decision could be based on the received signal strength, the existing traffic load on APs, or a combination of several metrics [15], [2]. Bejerano et. al. [3] proved that it is NP-complete to achieve global max-min throughput fairness under integral association control. They proposed approximation

TABLE I
NOTATIONS

Notations	Comments
M	Number of WiFi APs
N_{wifi}	Number of users in the WiFi-only network
N_{wimax}	Number of users in the WiMAX-only network
N_{int}	Number of users in the integrated network
N'_i	Number of users in AP i in the WiFi-only network
N_i	Number of users in AP i under virtual AP association
(i,j)	The jth user in AP i under virtual AP association
T_i	Throughput of both AP i and its members
L_i	Load of AP i
T_{wimax}	Throughput of WiMAX and its members
L_{wimax}	Load of WiMAX BS
r_{ij}	WiFi rate (i.e., average link capacity) of user (i,j)
R_{ij}	WiMAX rate (i.e., average link capacity) of user (i,j)
x_{ij}	Fraction of user (i,j)'s traffic to be sent in WiFi network
χ	Optimal fractional association from the LP

algorithms to guarantee the performance ratio to the optimal fractional association which is the fairest association possible. Our problem can be viewed as a special case of theirs (i.e., consider WiMAX BS as a special AP with a much larger transmission range to cover the whole network). However, we exploit the special structure of the integrated WiFi/WiMAX network, and propose an algorithm that is simpler and with better performance. The algorithm is also easier to implement because both its message exchange and the computation are distributed. The heuristic algorithm in [3], on the other hand, requires a central controller to gather global information, perform the computation, and disseminate the decision to each user.

III. NETWORK MODEL AND INTEGRATION GAIN

A. Network Model

We consider a service area large enough to contain multiple WiFi APs. For example, In Chicago, up to 256 APs can be found in 1/2 square mile suburban area [14]. Each AP has a limited transmission range, and only serves users within its range. Neighboring APs may have overlap in their coverage. We assume the whole service area is covered by these APs. There exists one WiMAX BS that also covers the whole area. It is a reasonable assumption since a

WiMAX BS can typically reach a distance up to tens of miles. All APs and the BS directly connect to the Internet. Each user is equipped with one WiFi radio and one WiMAX radio. Under the above coverage assumption, a user may hear one or more APs through its WiFi radio, and the BS through its WiMAX radio. It can choose to connect to a nearby AP, or the WiMAX BS, or even both by utilizing two radios at the same time. We use *integral* association to denote the first two cases because all traffic is sent on a single radio. We use *fractional* association to denote the last case because the user has to split its traffic on two active radios. In principle, fractional association provides better performance due to its flexibility. However, technical difficulties exist in practice. First, the requirement for the carrying device increases due to the excessive power consumption and heat when two radios are active at the same time. Second, the interference between the two co-located radios cannot be ignored even if they are operating on non-overlapping channels [19]. Last, protocol complexity increases dramatically due to traffic splitting and significant out-of-order packet delivery. Thus, we use the performance under the fractional association as the benchmark and study the integral association in practice.

We focus on the association decision between WiFi and WiMAX networks for each user. It is itself a challenging problem to determine which AP to associate with among nearby APs if a user decides to stay in the WiFi network. We assume there exists a rule to pre-determine an AP. The pre-determination rule could be any load balancing algorithm in WLAN [15], [2] or based on the received signal strength. Whenever the user decides to switch to the WiFi network, the WiFi radio always associates with the pre-determined AP. Given a set of users, we can determine the corresponding AP for each user following the pre-determination rule. We call such pre-determined user-AP mapping the *virtual AP association*, which is independent of the actual association. We use (i, j) to denote the jth user associated with AP i in the virtual AP association. Since a user can only associate with a single AP, (i, j) will be used to uniquely identify a user. It can help simply the formulations we will present later. It is obvious that different pre-determination rules have different impact on the system performance. A carefully designed rule should lead to a better performance than a random rule. But the integration gain will not be affected much as it is a relative performance metric. We will study it in more detail in Section VI. We use x_{ij} to denote the fraction of user (i, j)'s traffic to be sent through its WiFi radio, and $1-x_{ij}$ as the fraction of the traffic through its WiMAX radio. We have $x_{ij} \in [0, 1]$ in fractional association, and $x_{ij} \in \{0, 1\}$ in integral association. So the *actual* association of user

(i, j) is determined by x_{ij}.

We assume that the transmission in one AP does not interfere with that in adjacent APs. This can be achieved by assigning non-overlapping channels (e.g., 3 in 802.11b and 12 in 802.11a) to neighboring APs. WiMAX BS does not interfere with APs because it usually operates on a different frequency band. We use r_{ij} to denote the WiFi rate (i.e., link capacity) observed by user (i, j) in the long run, and R_{ij} the WiMAX rate. For example, a user may observe a WiFi rate of 54Mbps (e.g., 802.11a) under perfect channel condition, or lower than 54Mbps under significant path loss.

Within each AP and WiMAX BS, we assume the network is saturated, and the bandwidth is *fairly* shared among all associated users. Under saturated traffic, we note that WiFi MAC tries to evenly divides the access opportunity among its associated user in the long term [6], [5], which leads to roughly the same throughput for each user. We call it *throughput share*. On the other hand, WiMAX BS is fully responsible for allocating bandwidth for all users, in both the uplink and the downlink [1]. But the standard does not specify the scheduling algorithm, which is left for the system designer and developer to decide [9]. Therefore, WiMAX MAC can choose to achieve different bandwidth share objectives, including throughput share as WiFi MAC does. So in our work, we assume throughput share in an AP or the BS, i.e., users of the same AP or BS obtain the same throughput. We will consider other scheduling policies of WiMAX in future work. We use T_i to denote the throughput of each user in AP i. For abbreviation, we also call it the throughput of AP i. Assume we have N_i users in AP i under virtual AP association. Denote t_{ij} as the proportion of time for user (i, j) in AP i. We have $r_{ij}t_{ij} = r_{ik}t_{ik} \quad \forall j \neq k$. Given $\sum_{j=1}^{N_i} t_{ij} = 1$, we can obtain the throughput for each user in AP i under virtual AP association as,

$$r_{ij}t_{ij} = \frac{1}{\sum_{j=1}^{N_i} \frac{1}{r_{ij}}}. \tag{1}$$

Similar derivation can be applied on WiMAX BS. So under the actual association, the throughput of AP i is $T_i = \frac{1}{\sum_{j=1}^{N_i} x_{ij} \frac{1}{r_{ij}}}$. The throughput of WiMAX BS is $T_{wimax} = \frac{1}{\sum_i \sum_{j=1}^{N_i} (1-x_{ij}) \frac{1}{R_{ij}}}$.

We define "load" as the inverse of the throughput in an AP or the BS. For example, the load of AP i is $L_i = \frac{1}{T_i}$. In particular, user (i, j) contributes $x_{ij}\frac{1}{r_{ij}}$ amount of load to L_i, and $(1-x_{ij})\frac{1}{R_{ij}}$ amount of load to L_{wimax}. We list in Table I the important notations used throughout the paper.

B. A Generic Framework to Quantify Integration Gain

In this section, we propose a framework to quantify the integration gain, which has not been considered in prior work. We aim to make it generic so the framework can accommodate typical performance metrics such as minimum (average) throughput, maximum (average) delay, link quality, and reliability, etc. The framework consists of three steps:

1) Create a WiFi-only network with N_{wifi} users. Measure the network performance. We denote the performance as r_{wifi}. In this step, the network can be generated arbitrarily. We do not pose any control on it. For instance, it could be a randomly deployed WiFi network.
2) Create a WiMAX-only network with a controllable number of users N_{wimax} in the same service area. In this step, N_{wimax} is carefully adjusted to make the performance of the WiMAX network also r_{wifi}. For example, if the performance metric is average throughput, and $r_{wifi} = 0.5Mbps$, we can generate a set of WiMAX users and adjust its number so that the average throughput is close enough to $0.5Mbps$. Note that, under other metrics, it could be more complicated than simply adjusting the number of WiMAX users to achieve the same performance.
3) Integrate the two networks with their corresponding users, i.e., total number of users in the integrated network is $N_{int} = N_{wifi} + N_{wimax}$. Based on the first two steps, if the two networks are simply merged without interactions between them, the performance of the integrated network should still be r_{wifi}. On the other hand, there may exist metric-dependent interaction policies that improve the overall performance. We choose the best policy and denote its performance as r_{opt}. We define the integration gain as $\frac{|r_{opt}-r_{wifi}|}{r_{wifi}}$. If the best policy is impractical to find (e.g., the problem is NP-hard), one may resort to its approximations.

The key concept is as follows: *by ensuring WiMAX has the same performance as WiFi before the integration, we ensure that the gain comes from integration instead of additional resources.* On the other hand, the performance analysis of existing studies on integrated heterogeneous networks does not separate the integration from additional resources. In their analysis, the auxiliary network is added to the main network for free. The gain they observe results from a mixture of the two causes (i.e., integration and additional resources).

Before we can calculate the integration gain, we need to choose a performance metric, and derive its optimal policy. In this study, we focus on max-min throughput fairness. We choose max-min fairness because it improves worst-case experience and is achieved by the default WiFi access scheme. In addition, max-min fairness is more mathematically tractable, which enables us to focus on the essence of integration gain. We are aware that in a single-cell scenario, a user with a poor channel condition can deteriorate the performance of other users severely under max-min fairness. In this multi-AP WiFi/WiMAX network, the performance is determined by many factors, including user distributions, number of users in each AP, and network heterogeneity. Therefore, the impact of a single user is much smaller.

We also briefly cover the proportional fairness metric, which balances between the two competing objectives of maximizing the total throughput and providing a certain level of minimum throughput to the individual user. In the single-cell scenario, it is considered to be a better performance objective than max-min fairness because the throughput of each user is proportional to its data rate. In this work, we conduct a preliminary study on this metric to formulate the problem and provide some insights. A complete study on proportional fairness metric will be included in future work.

IV. Max-Min Throughput Fairness and Approximation Algorithm

We first prove it is NP-hard to achieve integral max-min throughput fairness in the integrated WiFi/WiMAX network. Then we propose an approximation algorithm that achieves guaranteed performance.

A. *Max-Min Throughput Fairness*

Let a throughput vector $\overrightarrow{T} = \{t_1, t_2, \cdots, t_N\}$ denotes the throughput distribution of all users in the network. Without loss of generality, we assume $t_i \leq t_j$ for $i \leq j$. Informally, max-min throughput fairness means that we *cannot* increase the throughput of one user without decreasing that of another user with equal or less throughput. Formally, it is defined as follows.

Definition 1: **Max-Min Throughput Fair**: A throughput vector $\overrightarrow{T}$ is called max-min fair if it has the highest lexicographical value among all throughput vectors. That is, if $\overrightarrow{T} \neq \overrightarrow{T'}$, there exists a position j such that $t_i = t'_i$ for $i < j$, and $t_j > t'_j$.

Note that "max-min" and "maximize the minimum" are two different concepts. We use the former to describe the bandwidth allocation with the best lexicographical order, and the latter to describe those with the maximum minimum throughput. So the former implies the latter, but the reverse usually does not hold. However, these two objectives are equivalent in an integrated WiFi/WiMAX network under fractional association. As we will show later, the corresponding fractional association can be easily obtained by solving a simple linear program (LP). On the other hand, to provide max-min fairness under integral association is NP-hard. We provide the proof below.

B. Proof of NP-hardness

We consider a special case of our problem: There are only one AP and one BS. Each user is within the coverage of both networks. Assume that the rate on the WiMAX link is the same as the WiFi link for a given user, but varies among users. We prove that this special case is NP-hard. The general case, where there are multiple APs and each user has different WiMAX and WiFi rates, is also NP-hard. We prove it by reducing *Partition* to our problem.

Definition 2: ***Partition*** (decision) : Can a set of numbers, S, be divided into two disjoint subsets S_1 and S_2, such that the sum of both subsets equals?

Let A be an instance of *Partition*. Each element in A has a weight associated with itself. Let the sum over all weights in A be $2D$. We then construct an instance of our problem B from A. We view each element in A as a user. The weight is the load contributed by this user. Under the previous assumption, each user contributes the same amount of load whether it stays in WiFi or WiMAX network. If A is a "Yes" instance of *Partition*, we can divide A into two subsets A_1 and A_2, each has a total weight of D. So we can also distribute the users in B such that the AP and the BS have the same load of D, and thus the same per-user throughput of $\frac{1}{D}$. It is the max-min throughput allocation. Conversely, the max-min throughput allocation of B could have two possibilities: 1) all users have the same throughput; 2) some users have different throughput than others. In case 1), the AP and the BS have the same load of D. Thus, we can follow the same user distribution to divide the elements in A into two subsets with the sum of weights equals D in each subset. A is thus a "Yes" instance. In case 2), the AP and the BS have different loads, and some users have throughput less than $\frac{1}{D}$. A must be a "No" instance. Otherwise, we can distribute the users in B such that all users have the same throughput of $\frac{1}{D}$. It is a throughput distribution of a higher lexicographical order than the given max-min throughput allocation, which is not possible.

C. Max-Min Fairness under Fractional Association

Optimal fractional association provides the best possible max-min throughput fairness and thus its performance serves as a benchmark for that of integral associations. While it is difficult to achieve max-min fairness under integral association, the optimal fractional association to achieve max-min fairness can be obtained by solving the following simple LP.

$$\begin{aligned} &\min \beta \\ &\sum_{j=1}^{N_i} x_{ij} \frac{1}{r_{ij}} \leq \beta \qquad \forall \text{ AP } i \\ &\sum_{i} \sum_{j=1}^{N_i} (1 - x_{ij}) \frac{1}{R_{ij}} \leq \beta \\ &0 \leq x_{ij} \leq 1 \qquad \forall\ (i,j). \end{aligned} \tag{2}$$

The objective is to minimize the maximum load among all users. If we let $\beta = \frac{1}{\alpha}$, where α represents the throughput, we see that it is equivalent to maximize the minimum throughput.

We denote the solution as χ. That is, χ is the vector to include the fraction x_{ij} for every user (i,j). Note that (2) does not have assumptions on initial conditions. So we should always get the same output no matter what the initial association the users may have. To ease the presentation, in the following discussion, we imagine all users initially associate with WiFi APs (i.e., following the virtual AP association). We say user (i,j) is *switched* to WiMAX under χ if $x_{ij} < 1$. It includes two cases: the user is in WiMAX entirely ($x_{ij} = 0$) or fractionally ($x_{ij} > 0$).

Given the association, we can compute the throughput for each user. We use $\vec{T}$ to denote the throughput distribution under χ. Now we prove that, $\vec{T}$ is max-min fair. First, we introduce the bottleneck group which is adapted from [3].

Definition 3: **Bottleneck group**: Under χ, WiFi APs with at least one user switched to WiMAX, together with WiMAX BS, are called the bottleneck group G_B.

Lemma 1: In $\vec{T}$, all users in the bottleneck group have the same throughput T, which is the inverse of the objective value of (2).

Proof: Define T as the inverse of the objective value of (2). Thus T is the minimum throughput in $\vec{T}$. Let us consider AP a in the bottleneck group. By definition, it has at least one user switched to WiMAX. We use T_a and T_{wimax} to denote the throughput of AP a and WiMAX, respectively. First we prove that T_{wimax} must equal to the minimum throughput of the network, which is T. Otherwise, there exists an AP with throughput $T_{min} < T_{wimax}$. Then we can switch some users from this AP to WiMAX until both reach the same throughput. Then T_{min} will be improved, which contradicts the objective of (2). Second, we prove $T_a = T_{wimax}$. Otherwise, we must have $T_a > T_{wimax}$. Then WiMAX can "return" some users it previously switched from AP a until T_a and T_{wimax} equal. It improves the minimum throughput of the network, contradicting (2).

The above proof can be applied to each AP in the bottleneck group. Thus, all users in the bottleneck group have the same throughput T. ■

Theorem 1: χ leads to the max-min throughput fairness under fractional association control.

Proof: We prove by contradiction. Assume we can find a better association χ', which leads to a better (in terms of lexicographical order) throughput distribution. Let $\vec{T}'$ denote the throughput vector under χ'. We have $\vec{T}' > \vec{T}$. From Lemma 1, the lowest throughput in $\vec{T}'$ must also be T, which is the best minimum throughput. Following the same proof as in Lemma 1, WiMAX must also have the lowest throughput under χ' . Let G denote the group of APs whose throughput are smaller than T before the load balancing. Under χ, each AP in G must have some users switched to WiMAX and thus belongs to the bottleneck group G_B. Under χ', each AP in G must have equal or larger throughput than T since $\vec{T}' > \vec{T}$. So these APs also have users switched to WiMAX. We argue that AP in G under χ' cannot have a throughput larger than T. Otherwise, such AP can increase the WiMAX throughput by recalling some original users from it. Then WiMAX can in turn help each AP with throughput of T a little bit by increasing the fraction of users it switches from these APs. The minimum throughput will be larger than T, which is not possible. Thus, each AP in G, and the WiMAX must also have the throughput of T under χ'. This suggests that exactly the same set of users with the same fraction are switched from G to WiMAX under both χ and χ'.

The only way $\vec{T}'$ can be better than $\vec{T}$ is therefore χ' may switch some users from APs with original throughput larger than T. These APs, denoted as G', do not have users switched to WiMAX under χ, thus remain the original throughput. So in $\vec{T}'$, at least one AP in G' has users switched to WiMAX. Then WiMAX must have a throughput less than T, which leads to $\vec{T}' < \vec{T}$.

■

Algorithm 1 Approximation

Each user query WiFi and WiMAX rates from its two radios, and report to its virtual AP
AP i sorts virtual users based on their WiMAX-WiFi rate ratio R_{ij}/r_{ij} in decreasing order
while The minimum virtual throughput improves **do**
  Find the AP with the minimum virtual throughput
  Switch the first user to WiMAX
  Check the virtual throughput of WiMAX and APs
end while
Output the association

Algorithm 2 Intermediate

$\chi^{frac} \leftarrow$ Solve LP (2)
Include every user j with $x_j = 1$ into WiMAX
For all the fractional users in χ^{frac}
while The minimum throughput improves **do**
  Find the AP with the minimum throughput
  Switch the fractional user to WiMAX
  Check the throughput of WiMAX and APs
end while

Algorithm 3 Reference paper

$\chi^{frac} \leftarrow Fractional\_Load\_Balancing(A, U)$
$\chi^{int} \leftarrow Rounding(\chi^{frac})$
return χ^{int}

D. Approximation Algorithm

Since our problem is NP-hard, we have to use an approximation algorithm to provide integral association in practice. We want the approximation algorithm to guarantee the performance relative to the optimal *fractional* solution, which is the fairest among all possible throughput distributions.

1) Algorithm Description: The algorithm is shown in Algorithm 1. It works on the integrated network with an arbitrary initial association. Each user queries the WiFi and WiMAX data rates from both radios (The WiFi rate is from the virtually associated AP). After each user reports its rate information to the corresponding virtual AP, each AP sorts all associated virtual users based on their WiMAX-WiFi rate ratio in a decreasing order. The algorithm then starts a loop. Inside the loop, the AP with the smallest throughput is selected, and its first user is marked to be switched to WiMAX. Each AP and the WiMAX BS then update their virtual throughput, and the algorithm starts the loop again until the minimum throughput of the integrated network stops increasing. A few observations are in order. The proposed algorithm exploits all three factors discussed earlier to achieve the network integration gain. The capacity ratio between WiMAX and WiFi interfaces (i.e., R_{ij}/r_{ij}) exploits multi-network diversity; the ranking among all users exploits multi-user diversity; and the selection of the AP with the lowest throughput exploits spatial multiplexing.

In practice, each AP can report its virtual throughput to WiMAX BS. The BS is then responsible for selecting the right AP in each iteration. The computation is distributed among APs and the BS. We have two layers of information exchange (i.e., user-AP and AP-BS) with limited message overhead in each layer. Due to the special structure of the integrated WiFi/WiMAX network, such a simple algorithm can still provide performance guarantee. In the following, we focus on the users switched to WiMAX under optimal fractional association χ. Among them, we define *integral* users as the users with $x_{ij} = 0$, and *fractional* users as the users with $0 < x_{ij} < 1$ under χ.

2) Proof of the Performance Bound:

Theorem 2: Algorithm 1 provides 2-approximation to the optimal fractional solution.

Proof: In the following, we outline the proof before we provide the details.

1) Prove that an intermediate algorithm (Algorithm 2) results in a throughput distribution with an equal or higher lexicographical order than an existing algorithm (Algorithm 3), which is shown to provide 2-approximation to the optimal fractional association.
2) Prove that our approximation algorithm (Algorithm 1) results in a throughput distribution with an equal or higher lexicographical order than the intermediate algorithm (Algorithm 2).

■

*a) **Step 1**:* Reference [3] presents a 2-approximation algorithm with threshold, as shown in Algorithm 3. In the algorithm, $Fractional\_Load\_Balancing(A, U)$ consists of two LPs and a simple graph coloring procedure. It gives the optimal fractional user association to provide max-min throughput fairness. The rounding method [16] constructs a bipartite graph based on the optimal fractional association, then uses maximal matching to determine the integral association. Since our problem can be viewed a special case of theirs in [3], this algorithm can also be applied on our problem after we replace $Fractional\_Load\_Balancing(A, U)$ with the LP defined in (2). Though applicable to the same problem, our algorithm is better than theirs because of two reasons. First, our algorithm can also provide 2-approximation to the optimal fractional solution. Numerical simulation (later in this section) shows that our algorithm outperforms theirs in practice. Second, the computation and information exchange of our algorithm are distributed, which makes our algorithm easier to implement. One important property of Algorithm 3 is that it switches all the integral users and a subset of fractional users to WiMAX. We need it for the following proof.

We design an intermediate algorithm shown in Algorithm 2. It takes χ as input. It first switches all integral users in χ to WiMAX. Then it performs a similar loop as in Algorithm 1. But it only looks at fractional users in each AP inside the loop. We now show that Algorithm 2 performs better than Algorithm 3 in terms of lexicographical value.

Lemma 2: Under χ, within each AP, a user switched with WiMAX has a higher or equal WiMAX-WiFi rate ratio than any user remaining in that AP.

Proof: The Lagrangian function of the LP defined in (2) is

$$\begin{aligned} L(\beta, \chi) \;\; &= \;\; \beta - \sum_{i=1}^{|A|} \theta_i \left(\beta - \sum_{j \in N_i} x_{ij} \frac{1}{r_{ij}} \right) - \sum_i \sum_{j=1}^{N_i} \lambda_{ij} x_{ij} - \sum_i \sum_{j=1}^{N_i} \omega_{ij} (1 - x_{ij}) \\ & \quad - \psi \left(\beta - \sum_i \sum_{j=1}^{N_i} (1 - x_{ij}) \frac{1}{R_{ij}} \right), \end{aligned}$$

where θ, λ, ω, and ψ are slack variables. According to Lagrangian Multiplier method and complementary slackness, we have the following equations,

$$\frac{\partial L}{\partial \beta} = 1 - \sum_{i=1}^{|A|} \theta_i - \psi = 0 \qquad (3)$$

$$\frac{\partial L}{\partial x_{ij}} = \theta_i \frac{1}{r_{ij}} - \lambda_{ij} + \omega_{ij} - \psi \frac{1}{R_{ij}} = 0$$
$$\lambda_{ij} x_{ij} = 0 \quad \forall \text{ user } (i,j)$$
$$\omega_{ij}(1 - x_{ij}) = 0 \quad \forall \text{ user } (i,j).$$

Within AP i, if user j has been switched, i.e., $x_{ij} < 1$, then we have

$$\omega_{ij} = 0, \lambda_{ij} \geq 0 \Rightarrow \theta_i \frac{1}{r_{ij}} \geq \psi \frac{1}{R_{ij}}.$$

Similarly, if user k has not been switched, i.e., $x_{ik} = 1$, then we have

$$\omega_{ik} \geq 0, \lambda_{ik} = 0 \Rightarrow \theta_i \frac{1}{r_{ik}} \leq \psi \frac{1}{R_{ik}}.$$

Thus we have $\frac{R_{ij}}{r_{ij}} \geq \frac{R_{ik}}{r_{ik}}$. ■

Corollary 1: Given χ, there exists an association χ' with the same performance where an AP can have at most one fractional user.

Proof: Under χ, if AP i has two fractional user (i,j) and (i,k), we have $0 < x_{ij}, x_{ik} < 1$. Thus $\frac{R_{ij}}{r_{ij}} = \frac{R_{ik}}{r_{ik}}$ based on (3). Similar idea applies to the case with multiple fractional users. So all fractional users in the same AP must have the same WiMAX-WiFi rate ratio. In this case, we can always "aggregate" multiple fractional users into some integral users and at most one fractional user without changing the performance. We start from a simple case by assuming AP a has 2 fractional users under χ. Their fractions are x_1 and x_2, WiFi rates are r_1 and r_2, and WiMAX rates are R_1 and R_2, respectively. The throughput of this AP is

$$T_a = \frac{1}{L_a + x_1 \frac{1}{r_1} + x_2 \frac{1}{r_2}}, \tag{4}$$

where L_a is the load contributed by other users in this AP. Similarly, the throughput of the WiMAX is

$$T_{wimax} = \frac{1}{L_{wimax} + (1 - x_1)\frac{1}{R_1} + (1 - x_2)\frac{1}{R_2}}, \tag{5}$$

where L_{wimax} is the load from other users in WiMAX. We have $\frac{R_1}{r_1} = \frac{R_2}{r_2}$. We consider two cases:

Case 1: $\frac{r_2}{r_1} x_1 + x_2 \leq 1$. We consider a new association χ' where both fractional users have associations of $x'_1 = 0, x'_2 = \frac{r_2}{r_1} x_1 + x_2$, and associations of other users remain unchanged. We can verify that the throughput of AP a and WiMAX is the same in χ and χ'. Since the throughput of other users remain the same, χ and χ' lead to the same throughput distribution.

Case 2: $\frac{r_2}{r_1}x_1 + x_2 > 1$. We consider a new association χ' where both fractional users have associations of $x'_1 = x_1 - (1-x_2)\frac{r_1}{r_2}, x'_2 = 1$. We have $0 < x'_1 \leq 1$. Similarly, χ and χ' lead to the same throughput distribution.

If we have multiple fractional users, we iteratively apply the same approach on two fractional users until at most one fractional user remains. ■

In the following, we assume each AP under χ has at most one fractional user. Otherwise, we can always use the corresponding χ' to replace χ.

Theorem 3: Algorithm 2 results in a throughput distribution with an equal or higher lexicographical order than Algorithm 3.

Proof: Note that Algorithm 3 switches all integral users and a subset of fractional users to WiMAX. From Corollary 1, Algorithm 2 also switches all integral users and a subset of fractional users to WiMAX. We use Γ to denote all algorithms which switch all integral users and a subset of fractional users to WiMAX. It suffices to prove that Algorithm 2 is the best in Γ in terms of max-min fairness. Suppose there exists Algorithm $2'$ in Γ with a better performance. Since they share the same set of integral users, they must differ in fractional users. Let Δ and Δ' be the set of fractional users switched in Algorithm 2 and Algorithm $2'$, respectively. It is trivial that Δ cannot be a subset of Δ'. Otherwise, the minimum throughput under Algorithm $2'$ will be lower than that of Algorithm 2. So let us focus on the case where some fractional users switched in Δ are not switched in Δ'. Consider one of these users, a, and the corresponding AP i it originally associated with. Let T_i^{a-} denote the throughput of AP i before a is switched with WiMAX. In Algorithm 2, the minimum throughput is strictly larger than T_i^{a-} because it keeps switching fractional users until the minimum throughput stops increasing. On the other hand, the minimum throughput of Algorithm $2'$ is at most T_i^{a-} because user a is not switched. Thus, Algorithm 2 actually performs better than Algorithm $2'$, which contradicts the assumption. ■

b) ***Step 2:*** Now we show that Algorithm 1 performs better than Algorithm 2. We first prove the following lemma.

Lemma 3: The set of users switched in Algorithm 1 is a subset of that under χ.

Proof: We prove by contradiction, and only need to consider the case where Algorithm 1 switches at least one user which is not switched under χ. We consider one of such user, a. We have two scenarios:

Scenario 1: a is from AP i which has no user switched under χ. So AP i must have an original throughput T_i larger than the bottleneck throughput T. If Algorithm 1 switches user a at some iteration, the minimum throughput in Algorithm 1 is larger than T_i. Thus, Algorithm 1 leads to a better max-min throughput distribution than that under χ, which is not possible.

Scenario 2: a is from AP i which has users switched under χ. Thus, the throughput of AP i under χ is T. From Lemma 2, we know that users switched under χ have a larger WiMAX-WiFi rate ratio than users stay associated with the original AP. Since Algorithm 1 sorts users based on their rate ratio, user a must have a lower ratio than users switched under χ. Thus, switching user a suggests that users switched under χ should have already been switched in Algorithm 1. Therefore AP i already has a throughput at least T before a is switched. Then after a is switched in Algorithm 1, the minimum throughput will be larger than T. We face the same contradiction as in the first scenario. ■

Theorem 4: Algorithm 1 results in a throughput distribution with an equal or higher lexicographical order than Algorithm 2.

Proof: Let Ω denote the set of users Algorithm 1 switches. By Lemma 3, Ω is a subset of all users switched under χ. We have two scenarios.

Scenario 1: Ω includes all integral users and a subset of fractional users under χ. In this case, Algorithm 1 and Algorithm 2 perform exactly the same.

Scenario 2: Ω omits at least one integral users under χ, and includes the other integral users and a subset of fractional users. We consider one of the omitted integral user a from AP i. Let T_i^{a-} denote the throughput of AP i before a is switched in Algorithm 1. We assume T_i^{a-} is the smallest throughput among all APs containing omitted integral users. There must exist at least one fractional user which is switched to WiMAX. Otherwise, Algorithm 1 will not stop because WiMAX still has a higher throughput than the bottleneck throughput T (no fractional user in WiMAX yet). It can therefore switch user a to WiMAX to achieve a better throughput distribution. We look at the *last* fractional user b to be switched to WiMAX under Algorithm 1. Assume the corresponding AP is j. According to Algorithm 1, we have $T_j^{b-} < T_i^{a-}$. We prove by contradiction. Suppose Algorithm 2 performs better. We argue that the fractional user b should also be switched in Algorithm 2. Otherwise the minimum throughput under Algorithm 2 is at most T_j^{b-} while the minimum throughput is strictly larger than T_j^{b-} under Algorithm 1. We can argue the same on all the fractional users switched by Algorithm 1 before b because the corresponding APs have throughput less than T_j^{b-} before their fractional users are switched. These users should also be switched in Algorithm 2. Since Algorithm 2 switches an extra user a to WiMAX, its WiMAX throughput should be lower than that of Algorithm 1.

If WiMAX throughput is the lowest in both algorithms, we have a contradiction that Algorithm 1 actually performs better than Algorithm 2. Otherwise, there must be an AP k which has the lowest throughput under Algorithm 1, while under Algorithm 2, AP k has equal or better throughput. But it cannot be better because it means the fractional user is switched to WiMAX under Algorithm 2. Since WiMAX throughput under Algorithm 1 is higher, it can also switch the fractional user in AP k before the algorithm stops. We argue the same for each AP except for AP i because it has a lower throughput in Algorithm 1 than in Algorithm 2. But WiMAX throughput under Algorithm 1 has to be lower than T_i^{a-}, otherwise it can always switch a to WiMAX. Thus, if WiMAX is not the lowest, the throughput vectors before WiMAX must be equal under both algorithms, and they differ from WiMAX. Thus, we have the same contradiction. ■

V. Proportional Fairness

In this section, we define the proportional fairness metric and provide the formulation of the problem. As will be shown later, the objective function of proportional fairness is nonlinear and non-convex. Due to its inherent complexity, neither the optimal policy nor an approximate algorithm for the proportional fairness metric can be easily derived. We apply the Lagrangian Multiplier method on the formulation to gain some insights. We infer the optimal policy for a special scenario, which motivates us to design a heuristic for the general scenario.

Basically, proportional fairness allocates bandwidth to users in proportion to their data rates. Formally, it is defined as follows.

Definition 4: **Proportional Throughput Fair**: A throughput vector $\overrightarrow{T}$ is called proportionally fair if the product of all individual throughput components is the maximum among all throughput vectors. That is, $\overrightarrow{T}$ is the solution to argmax $\prod_{i=1}^{n} t_i$.

According to the definition, the optimal user association to achieve proportional fairness can be obtained by solving the following optimization problem,

$$\max_{\vec{x}} \left\{ \left(\prod_{i=1}^{M} \frac{1}{\left(\sum_{j=1}^{N_i} x_{ij} \frac{1}{r_{ij}}\right)^{\sum_{j=1}^{N_i} x_{ij}}} \right) \times \frac{1}{\left(\sum_{i=1}^{M} \sum_{j=1}^{N_i} (1 - x_{ij}) \frac{1}{R_{ij}}\right)^{\sum_{i=1}^{M} \sum_{j=1}^{N_i} (1-x_{ij})}} \right\}. \tag{6}$$

Inverse the formulation and take the logarithm form, we have an equivalent formulation,

$$\min_{\vec{x}} \left\{ \left(\sum_{i=1}^{M} \left(\sum_{j=1}^{N_i} x_{ij} \log \sum_{j=1}^{N_i} x_{ij} \frac{1}{r_{ij}} \right) \right) + \sum_{i=1}^{M} \sum_{j=1}^{N_i} (1 - x_{ij}) \log \left(\sum_{i=1}^{M} \sum_{j=1}^{N_i} (1 - x_{ij}) \frac{1}{R_{ij}} \right) \right\}. \tag{7}$$

We apply the Lagrangian Multiplier method. The Lagrangian function of (7) is,

$$\begin{aligned} L(\vec{x}, \vec{\lambda}, \vec{\omega}) = & \sum_{i=1}^{M} \left(\left(\sum_{j=1}^{N_i} x_{ij} \right) \log \sum_{j=1}^{N_i} x_{ij} \frac{1}{r_{ij}} \right) + \left(\sum_{i=1}^{M} \sum_{j=1}^{N_i} (1 - x_{ij}) \right) \log \left(\sum_{i=1}^{M} \sum_{j=1}^{N_i} (1 - x_{ij}) \frac{1}{R_{ij}} \right) \\ & - \sum_{i=1}^{M} \sum_{j=1}^{N_i} \lambda_{ij} x_{ij} - \sum_{i=1}^{M} \sum_{j=1}^{N_i} \omega_{ij} (1 - x_{ij}). \end{aligned} \tag{8}$$

Including the complementary slackness, for each user (i, j), we have,

$$\begin{aligned} & \frac{\partial L}{\partial x_{ij}} = \log \sum_{j=1}^{N_i} x_{ij} \frac{1}{r_{ij}} + \frac{\frac{1}{r_{ij}} \sum_{j=1}^{N_i} x_{ij}}{\sum_{j=1}^{N_i} x_{ij} \frac{1}{r_{ij}}} - \log \sum_{i=1}^{M} \sum_{j=1}^{N_i} (1 - x_{ij}) \frac{1}{R_{ij}} \\ & \qquad - \frac{\frac{1}{R_{ij}} \sum_{i=1}^{M} \sum_{j=1}^{N_i} (1 - x_{ij})}{\sum_{i=1}^{M} \sum_{j=1}^{N_i} (1 - x_{ij}) \frac{1}{R_{ij}}} - \lambda_{ij} + \omega_{ij} = 0 \\ & \lambda_{ij} x_{ij} = 0 \\ & \omega_{ij} (1 - x_{ij}) = 0. \end{aligned} \tag{9}$$

Now we consider two users (i, j) and (i, k), i.e., the jth and kth user in AP i under virtual association. Assume the optimal policy to achieve proportional fairness switched (i, j) to WiMAX while leave (i, k) in AP i. Thus, we must have $\omega_{ij} = 0$ and $\lambda_{ik} = 0$. As slack variables, $\lambda_{ij} \geq 0$ and $\omega_{ik} \geq 0$, then we have,

$$\frac{\left(\frac{1}{r_{ij}} - \frac{1}{r_{ik}}\right) \sum_{m=1}^{N_i} x_{im}}{\sum_{m=1}^{N_i} x_{im} \frac{1}{r_{im}}} \geq \frac{\left(\frac{1}{R_{ij}} - \frac{1}{R_{ik}}\right) \sum_{i=1}^{M} \sum_{m=1}^{N_i} (1 - x_{ij})}{\sum_{i=1}^{M} \sum_{m=1}^{N_i} (1 - x_{ij}) \frac{1}{R_{im}}}. \tag{10}$$

The optimal policy cannot be directly obtained from (10). We consider a special case where both users have the same WiMAX rate, i.e., $R_{ij} = R_{ik}$. In this case, we must have $r_{ij} \leq r_{ik}$ to satisfy (10). So the optimal policy under this special case must always switch the user with the

Algorithm 4 Heuristic for proportional fairness

Each user query WiFi and WiMAX rates from its two radios, and report to its virtual AP
AP i sorts virtual users based on their WiMAX-WiFi rate ratio R_{ij}/r_{ij} in decreasing order
while The product of the virtual throughput from all users improves **do**
  Save the product of throughput as previous product
  for all AP i **do**
    Temporarily switch its first user to WiMAX
    Calculate the current product of throughput from all users
    Record the change in the product from previous one
  end for
  Select the AP which leads to the highest change in product and switch its first user to WiMAX
end while
Output the association

smallest WiFi rate within an AP. Note that, it is also a special case of switching the user with the highest WiMAX/WiFi rate ratio where users have the same WiMAX rate. In general, when users have non-uniform WiMAX rates, we conjecture that the idea of switching the user with the highest WiMAX/WiFi rate ratio may still achieve a good performance. Thus, we propose a heuristic algorithm to achieve proportional fairness in the integrated WiFi/WiMAX network (i.e., Algorithm 4). Similar to the heuristic under max-min fairness, Algorithm 4 also sorts users within an AP based on their WiMAX/WiFi rate ratio. The difference is, in each iteration, Algorithm 4 compares all APs, and switch the user from the AP which leads to the highest increase in throughput product. While we do not claim that Algorithm 4 achieves a guaranteed performance ratio, simulation results show that it achieves good performance compared to the optimal solution. We shown it in next section.

VI. Performance Evaluation through Numerical Simulations

We use Matlab to conduct numerical simulations. We assume IEEE 802.11a as the MAC and physical layer standard for WiFi. The channel bandwidth is 20Mhz for WiFi and 10Mhz for WiMAX. The transmission power is set as 40mW and 80mW for WiFi and WiMAX, respectively. We assume both WiFi and WiMAX use OFDM with adaptive modulation. They differ in symbol rate, number of carriers, and coding rates. We adopt the value and formulations suggested by the standard or in the literature [1], [8]. The supported modulation schemes include QAM64, QAM16, QPSK and BPSK. We start from QAM64, calculate the corresponding data rate and

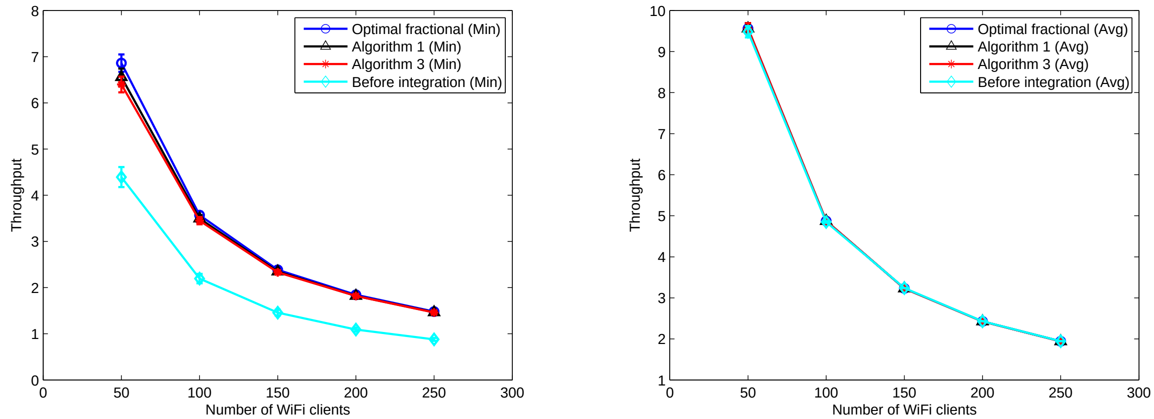
BER. If the BER exceeds a pre-defined target BER (e.g., $1e^{-5}$), we switch to the next modulation scheme which leads to lower data rate and BER. We repeat this process until we meet the BER requirement or we reach the last modulation scheme. In the latter case, we claim the link is broken. Otherwise, we use the corresponding data rate and BER. Once the data rates are determined, we use the simplified "throughput share" link-layer model within each AP and the WiMAX BS to determine the throughput of each user (see Section III-A for details).

Using MATLAB-based simulations, we have tried to capture the impact of the aspects of MAC that significantly impact our study. A more detailed simulation (e.g., in ns-2) would certainly help, but would not changes the inferences of the study. We include the use of a detailed simulation environment in our future work.

A. Max-Min Fairness

We use simulations to compare the performance of the optimal fractional solution, the proposed heuristic algorithm (i.e., Algorithm 1), and the algorithm in [3] (i.e., Algorithm 3). We first study the integration gain by comparing the three algorithms with the performance of the separate network before integration. We then investigate the impact of the virtual AP association policy on the performance of the heuristic algorithm.

1) Integration Gain: We consider a service area of 1500x1500. Nine APs form a regular grid in the service area, while users are randomly and uniformly distributed. We assume that users determine the virtual AP association based on the received signal strength. We name it *nearest AP* policy. WiMAX BS is placed at the center of the service area. We change the number of users from 50 to 250. For a given number of clients, we average the results over 50 random instances and plot the confidence interval. In each instance, we follow the three steps in the generic framework. We use the minimum throughput across the network as the performance metric. Recall that the throughput vector is sorted in ascending order. According to Definition IV-A, a better minimum throughput is *sufficient* to guarantee a better throughput vector in terms of max-min throughput fairness. We plot the throughput under three algorithms (i.e., optimal, Algorithm 1 and Algorithm 3) in the integrated network and that of the two networks before integration. Note that, the WiMAX network introduces additional resource but also *additional users* such that the WiMAX network has the same performance as the WiFi network. So the performance curve of "before integration" represents the performance of the two individual networks without



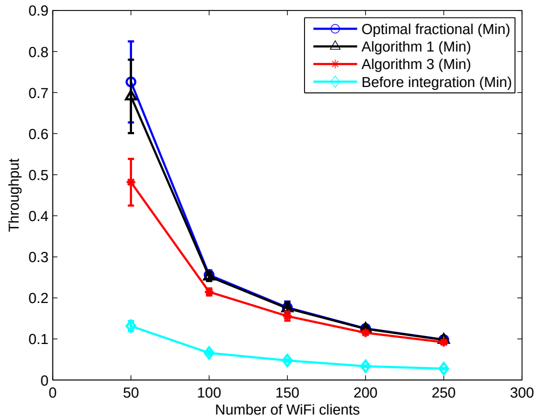
(a) Uniform rate: Minimum throughput

(b) Uniform rate: Average throughput

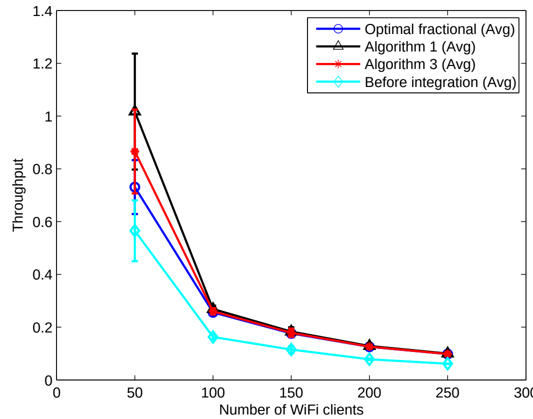
Fig. 1. Spatial multiplexing gain: uniform rate scenario

interactions between them. In the following, we design different simulation scenarios to focus on three aspects of the proposed algorithm that correspond to the three types of gains we discussed before, respectively.

a) Spatial Multiplexing Gain: In order to separate spatial multiplexing gain from others, we study the scenario where users have uniform WiFi and WiMAX rates. Uniform rate means that all users in the same network experience the same rate, e.g., 54Mbps in WiFi and 50Mbps in WiMAX. In this case, all users associated with the same AP are equivalent (i.e., has the same WiMAX/WiFi rate ratio). The proposed algorithm does not distinguish between users. It is thus reduced to perform the load balancing between WiFi and WiMAX network by switching a certain number of users. Fig. 1(a) shows the minimum throughput under uniform rate. In this case, Algorithm 1 performs slightly better than Algorithm 3. Both of them achieve close to optimal performance. We observe about 60% integration gain, which comes from the load balancing. Users in congested WiFi APs will be switched to WiMAX to improve the minimum throughput. However, as shown in Fig. 1(b), the integrated network has nearly the same average throughput as before integration. Note that spectrum efficiency determines the average throughput. Before integration, the two networks have the same average throughput. Switching users does not improve spectrum efficiency when users in an AP or the BS have the same spectrum efficiency (i.e., data rate). So the integration gain only comes from spatial multiplexing.



(a) Non-uniform rate: Minimum throughput



(b) Non-uniform rate: Average throughput

Fig. 2. Multi-network and multi-user diversity gain: non-uniform rate scenario

b) Multi-network and Multi-user Diversity Gain: Next, we consider the scenario where users have non-uniform data rates. The data rate is determined by the received SNR, the target bit error rate (BER) and the corresponding modulation scheme. In general, the rate decreases with the distance between the sender and the receiver. Under non-uniform rate, we have difficulties in generating the WiMAX network in step two of the framework. In this case, the minimum throughput is determined by both the number of users in the group and the minimum data rate among users. It is difficult to create the WiMAX network with the same minimum throughput because of the variations in the data rate. So we make a slight modification to the framework. We generate the WiMAX network based on the average throughput. But the integration gain is still calculated based on the minimum throughput. The performance trend should be the same as in the original framework.

Fig. 2(a) plots the minimum throughput under non-uniform rate. Algorithm 1 performs close to optimal, and considerably outperforms Algorithm 3. The gain is about 250%, which is much larger than that under uniform rate. This is because two additional sources of integration gain exist in this scenario. Under the non-uniform rate case, users with low WiFi rates may have high WiMAX rates or vice versa. By switching these users from where it has low rate to where it has high rate, in addition to the load balancing, the spectrum efficiency also improves, which is indicated by higher average throughput in the integrated network than that of before integration (Fig. 2(b)). In other words, we exploit the *multi-network diversity*. Furthermore, among all users

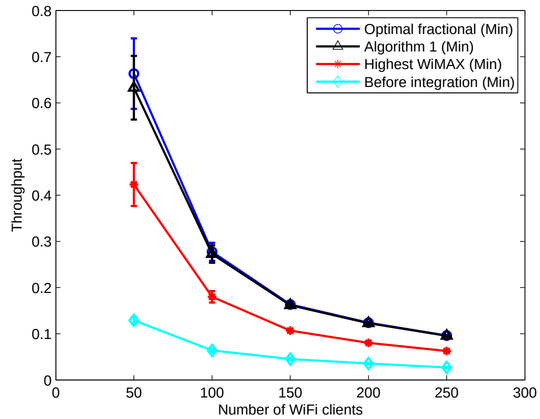


Fig. 3. Separate multi-user diversity from multi-network diversity: comparison between the proposed algorithm and an intermediate algorithm

which are available to be switched to WiMAX, the heuristic algorithm always switches the users with the highest WiMAX-WiFi rate ratio, which is to exploit the *multi-user diversity*. These are the reasons for the large improvement we observe. We also observe that the optimal fractional solution leads to a lower average throughput than Algorithm 1 and Algorithm 3, which is not surprising. To improve the minimum throughput, the optimal solution tends to allocate more time to the users with low data rates, which leaves less time for users with higher data rates. Thus the average throughput hurts. In addition, we see that Algorithm 1 outperforms Algorithm 3 in both minimum and average throughput.

To further separate multi-user and multi-network diversity gain, we compare the proposed algorithm with an intermediate algorithm, called *Highest WiMAX*. It differs from the proposed algorithm in only one aspect: *Highest WiMAX* sorts users based on WiMAX rate only rather than WiMAX/WiFi rate ratio. So it switches the users in a different order than the proposed algorithm. Fig. 3 shows the performance of both algorithms. It clearly indicates that the order of switching users also plays an important role in determining the integration gain we can achieve.

2) Impact of Virtual AP Association Policy: We compare the performance of the proposed heuristic algorithm under different virtual AP association policies. Besides the *nearest AP* policy, we also implement a simple *random* policy. That is, users randomly choose an AP among all APs that can be heard. We increase the number of APs to 16 to ensure that most users have more than one AP to choose from. We assume users have non-uniform data rates. As can be

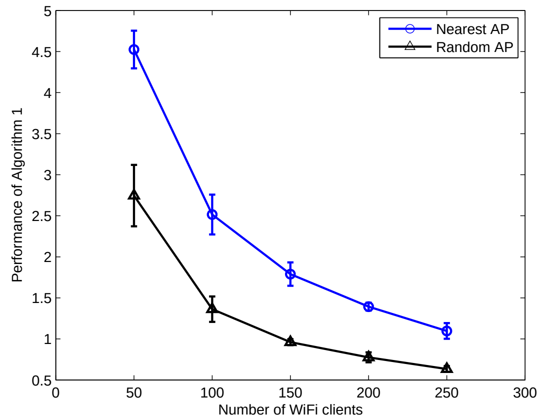


Fig. 4. Performance of Algorithm 1 under different virtual AP association policies.

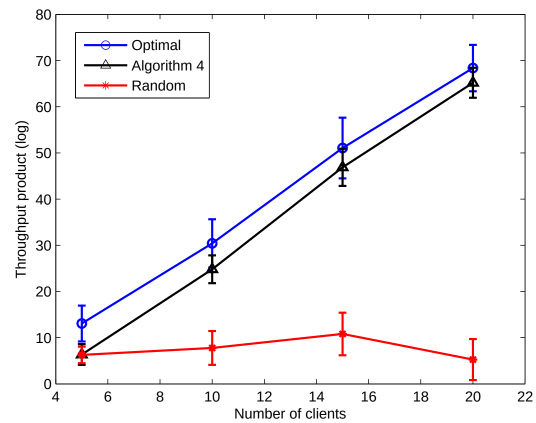


Fig. 5. Proportional fairness: Througput product (log) of three algorithms: optimal solution, the proposed heuristic (Algorithm 4), and a simple random algorithm.

imagined, users are more evenly distributed under *random* policy. But the average data rate is also lower.

As shown in Fig. 4, Algorithm 1 reaches a higher throughput performance under *nearest AP* policy than under *random* policy. Under *random* policy, users tend to have lower data rates in APs, which limit the maximum minimum throughput across the system. On the other hand, we still achieve an integration gain of around 230%, which is comparable to that under *nearest AP* policy. In summary, change in virtual AP association has an impact on the absolute performance. But it does not affect the integration gain much since it is a relative performance metric.

B. Proportional Fairness

In this work, we show the preliminary simulation results to validate the performance of the proposed heuristic algorithm. We compare it with the optimal solution and a random algorithm which simply select a random user from WiFi network to switch to WiMAX until the throughput product stops increasing. We use brute force search to obtain the optimal solution. Due to the exponential complexity, we cannot compute the optimal solution for network with more than 20 users. We focus on the non-uniform rate scenario.

Fig. 5 plots the throughput product of the three algorithms. The proposed heuristic achieves close-to-optimal performance in terms of the throughput product. Under our heuristic, users with higher WiMAX-WiFi rate ratio will be switched earlier. That is, our heuristic tends to replace a small throughput with a large one by switching a user, which is an intuitive way to improve the throughput product of the system. It also explains the big gap between the random algorithm and the other two. In the future, we will perform a deeper investigation on the framework and the heuristic to obtain more insights.

VII. CONCLUSIONS

In this paper, we study the integration gain of integrated WiFi/WiMAX network. Previous work on integrated heterogeneous networks usually assumes one of the networks is the main, and compare the performance of the integrated network with the main network. Thus the performance gain comes from the additional resources brought by the auxiliary network as well as the network integration. To our knowledge, we are the first to propose a framework to explicitly identify the integration gain, which is separated from the impact of additional resources. In other words, we quantify the gain from the network heterogeneity and better resource utilization. The framework supports different performance metrics. In this study, we focus on max-min throughput fairness and briefly cover the proportional throughput fairness. The proposed framework does not depend on any specifics of WiFi or WiMAX. In fact, it can be applied to any integrated heterogeneous wireless networks. The optimal policy in step three, however, does depend on the actual protocols of the two networks.

We prove that it is NP-hard to achieve integral max-min fairness. We propose a heuristic algorithm that provides 2-approximation to the optimal fractional association policy. The algorithm is simple and intuitive. It is also easy to implement due to its distributed nature. Numerical simulations show significant gain under both uniform and non-uniform rate scenarios. We identify three sources of integration gain, namely the spatial multiplexing, multi-network diversity, and multi-user diversity.

For the proportional fairness metric, we derive the formulation and propose a heuristic algorithm. The proposed algorithm achieves close-to-optimal performance in simulations.

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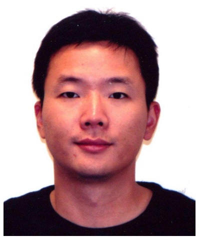

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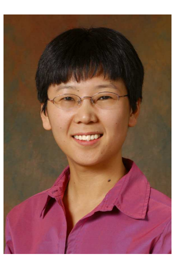

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