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Real-time scheduling method for networked discrete control systems

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ABSTRACT

This paper proposes a new scheduling method to obtain a maximum allowable delay bound for a scheduling of networked discrete control systems. The proposed method is formulated in terms of linear matrix inequalities (LMI) and can give a much less conservative delay bound than the existing methods. An event based network scheduling method is presented based on the delay bound obtained through the proposed method, and it can adjust the sampling period to allocate identical utilization to each control loop. The presented method can handle sporadic emergency data, periodic data, and non-real-time data.

1. Introduction

In distributed control systems (DCSs), a feedback control loop is closed through a network. The DCSs with networks are called networked control systems (NCSs). In an NCS, various delays occur due to sharing a common network medium, which are called network-induced delays (Asok & Yoram, 1988; Krtolica et al., 1994). Network-induced delays can vary widely according to the transmission time of messages and the overhead time. The performance of the control system is assumed to be affected by network-induced parameters such as delays, jitters, packet losses and link failures (Vatanski, Georges, Aubrun, Rondeau, & Jamsa-Jounela, 2008). The network in the NCS should handle three types of data: sporadic emergency data, periodic data, and non-real-time message. The transmission time through the media is largely dependent on the network protocols, especially data link layer protocols of networks and data length. Hence, it is necessary to present the methods to make these network-induced delays bounded and smaller, which are called network scheduling methods for the NCS.

In general, a faster sampling rate is said to be desirable in sampled-data systems so the discrete-time control design and performance can approximate that of the continuous-time system. But in NCSs, a faster sampling rate bound can increase network load, which in turn results in longer delay of the signals. Thus finding a sampling rate that can both tolerate the network-induced delay and achieve desired system performance is important in NCS design. This certain bound is called a maximum allowable delay bound (MADB) of the NCS.

Therefore, it is necessary to find the MADB for stability of the NCS, and then to find an appropriate network scheduling method that limits the network-induced delay to less than the MADB. A network scheduling method is required to reduce network-induced delays within the MADBs, while guaranteeing real-time transmission of sporadic, periodic data, and minimizing network utilization for non-real-time message.

The MADB has been obtained from stability conditions of control systems. There have been some results on the stability of NCSs (Lian, Moyne, & Tilbury, 2002; Yoram & Asok, 1988). Less conservative results on the MADB in non-NCSs are reported in Li and de Souza (1997a, 1997b) and Park (1999). In these papers, the MADB is obtained using the Ricatti equation approach, which yields conservative delay bounds. A scheduling method was presented in the NCS with Fieldbus networks (Beauvais & Deplanche, 1995; Cavalieri, Stefano, & Mirabella, 1995). But those papers did not consider the MADB, which were important in control applications.

There have been some studies on scheduling algorithms that can be applied to the NCS (Hong, 1995; Raja, Vijayananda, & Decotignie, 1993; Walsh, Hong, & Bushnell, 1999; Zhang, Branicky, & Phillips, 2001). A dynamic scheduling algorithm modified from the rate monotone scheduling algorithms was presented for periodic and sporadic data in Fieldbus networks (Raja et al., 1993). A heuristic algorithm was presented for periodic tasks only (Beauvais & Deplanche, 1995), but it did not support sporadic data. The several algorithms for dynamically scheduling of NCSs were proposed (Hong & Walsh, 2001; Zuberi & Shin, 1997). It had limitations when applied to the NCS because it did not consider some characteristics of the NCS, such as the MADB and sampling

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periods. A scheduling algorithm that can allocate the bandwidth of a network and determine sensor data sampling periods was presented by Hong (1995). In Hong (1995), the control system had only single input and single output (SISO), only periodic data were considered, and the MADB was not obtained analytically.

A network scheduling method considering three types of data based on a multi-input and multi-output (MIMO) system was proposed by Park, Kim, Kim, and Kwon (2002). In this paper, the estimation of MADB using the Ricatti equation is too conservative, which means the estimated MADB is too small and the network scheduling method discussed is somewhat heuristic.

In Branicky, Phillips, and Zhang (2000) and Walsh and Hong (2001), calculation methods of MADBs and stability analysis of NCSs were presented. However, these results were conservative to be of practical use and still remains to be improved. Further research is needed with regard to estimation of a less conservative MADB for stability of the NCS and systematic scheduling methods for three types of data.

In Kim, Lee, Kwon, and Park (2003), a calculation method of MADBs of NCS based on continuous-time was presented in terms of linear matrix inequalities (LMI). This method gave a much less conservative delay bound than previous methods. However, a calculation method of MADBs based on continuous model and a scheduling method for three types of data remain to be improved.

This paper proposes a method to obtain the MADB guaranteeing a stability of the discrete-time NCS. Using obtained MADBs, sampling period decision and bandwidth allocation method are presented. A proposed scheduling method can adjust the sampling period to allocate the same utilization to each control loop. It can handle three types of data and guarantees real-time transmission of sporadic emergency and periodic data. It is modified earliest deadline first (EDF) scheduling method which give priority to sporadic emergency data.

This paper is organized as follows. In the following section, an discrete-time NCS model is described. The MADB for the stability of the NCS is derived by LMI formulation. In Section 3, a network scheduling method that allocates the bandwidth and determines the sampling period for the NCS is presented. In Section 4, simulation results are given to show that method is useful. Finally, the conclusions are presented in Section 5.

2. MADB for stability in a control loop

In general, NCSs can be described as Fig. 1 (Nilson, 1998; Nilson, Bermhardsson, & Wittermark, 1998). A networked control loop is composed of a controller, sensors, and actuators through a common communication medium.

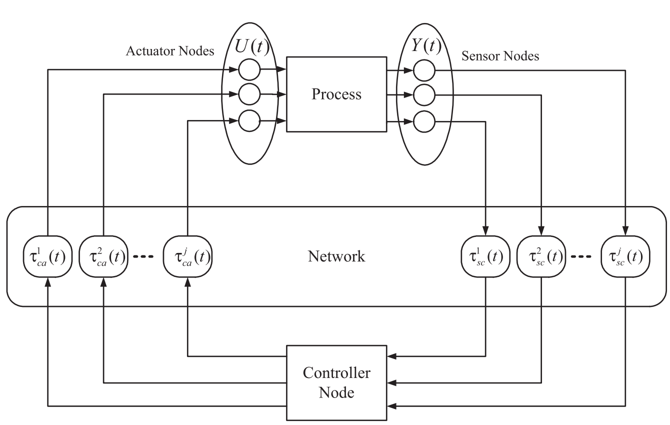

Fig. 1. Networked control loops with sensors and actuators.

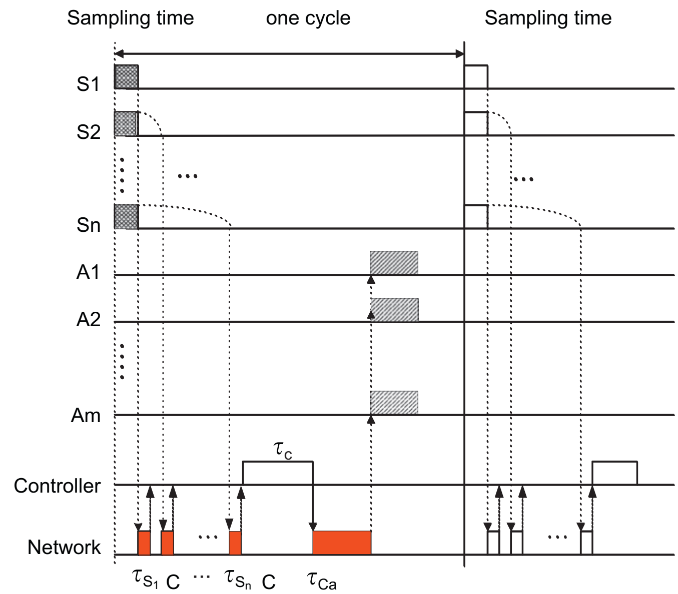

Fig. 2. Timing diagram of signals in the NCS.

In Fig. 2 (Kim et al., 2003), the timing diagram illustrates the process output and sampling instants, the signal into the controller node, the signal into the actuator node and the network-induced delay.

The MADB is defined as the maximum allowable interval from the instant when sensor nodes sample sensor data from a plant to the instant when actuators output the transferred data to the plant. If the sampling period in the jth loop exceeds the given MADB, then stability of the NCS could not be guaranteed. In this case, the outputs of the plant could deviate from the desired trajectory, or the controlled system. Hence, it is necessary to derive the MADB from parameters and configurations of the given plant and the controller.

In this paper, a stability is checked by single control loop model with each sensor and actuator node. The node which have a multiple control loop can be changed to the sum of nodes have a single control loop. That is to say, the node which have a multiple control loop can be changed to the sum of nodes which have a single control loop.

2.1. MADB in discrete-time system

A plant in a single control loop can be described in the following discrete-time state space form:

$$x_p(k+1) = A_p x^k + B_p \bar{u}(k),$$
$$y(k) = C_p x^k, \tag{1}$$

where $\bar{u}(k) \in R^{N_A}$, $x_p(k) \in R^{N_P}$, $y(k) \in R^{N_S}$. N_A, N_S, and N_P is the dimension of the actuators, sensors, and plant in the control loop. A_p, B_p, and C_p are matrices of appropriate sizes.

A controller in the control loop j can be described by

$$x_c(k+1) = A_c x_c(k) + B_c \bar{y}(k),$$
$$u(k) = C_c x_c(k) + D_c \bar{y}(k), \tag{2}$$

where $u(k) \in R^{N_A}$, $x_c(k) \in R^{N_C}$, $\bar{y}(k) \in R^{N_S}$. N_C is the dimension of the controller in the control loop.

$y(k)$ is the output of the plant and $\bar{y}(k)$ is the input to the controller. Similarly, $u(k)$ is the output of the controller and $\bar{u}(k)$ is the input to the plant. Let the step number of transmission from sensor to controller is τ_{sc}, which satisfies $0 \leqslant \tau_{sc} \leqslant \bar{\tau}_{sc}$ and the step number of transmission from controller to plant is τ_{cp}, which including the time from controller to actuator and other required time, and also satisfies $0 \leqslant \tau_{cp} \leqslant \bar{\tau}_{cp}$.

The communication delays are modeled as

$$\begin{aligned}\bar{y}(k) &= y(k-\tau_{sc}),\\ \bar{u}(k) &= u(k-\tau_{cp}),\\ \tau_{cp} &= \tau_{ca}+\tau_{other}.\end{aligned} \tag{3}$$

Using the above equations, a discrete-time NCS model becomes

$$\begin{aligned}x_p(k+1) &= A_p x_p(k) + B_p u(k-\tau_{cp})\\ &= A_p x_p(k) + B_p[C_c x_c(k-\tau_{cp}) + D_c \bar{y}(k-\tau_{cp})]\\ &= A_p x_p(k) + B_p[C_c x_c(k-\tau_{cp}) + D_c y(k-\tau_{cp}-\tau_{sc})]\\ &= A_p x_p(k) + B_p[C_c x_c(k-\tau_{cp}) + D_c C_p x_p(k-\tau_{cp}-\tau_{sc})].\end{aligned} \tag{4}$$

Similarly,

$$\begin{aligned}x_c(k+1) &= A_c x_c(k) + B_c y(k-\tau_{sc})\\ &= A_c x_c(k) + B_c C_p x_p(k-\tau_{sc}).\end{aligned} \tag{5}$$

Let $x(k) = [x_p(k) x_c(k)]^{\mathrm{T}}$, then Eqs. (4) and (5) become

$$\begin{aligned}x(k+1) &= \begin{bmatrix} x_p(k+1)\\ x_c(k+1)\end{bmatrix}\\ &= Ax(k) + A_1 x(k-\tau_1) + A_2 x(k-\tau_2) + A_3 x(k-\tau_3),\end{aligned} \tag{6}$$

where

$$A = \begin{bmatrix} A_p & 0\\ 0 & A_c\end{bmatrix},\quad A_1 = \begin{bmatrix} 0 & B_p C_c\\ 0 & 0\end{bmatrix},\quad A_2 = \begin{bmatrix} 0 & 0\\ B_p C_p & 0\end{bmatrix},\quad A_3 = \begin{bmatrix} B_p D_c C_p & 0\\ 0 & 0\end{bmatrix},$$

$$\tau_1 = \tau_{cp},\quad \tau_2 = \tau_{sc},\quad \tau_3 = \tau_{cp}+\tau_{sc}.$$

Eq. (6) presents each control loop in the discrete-time NCS using three types of delays. To generalize results to the multiple state-delayed case, consider the following system:

$$\begin{aligned}x(k+1) &= Ax(k) + \sum_{i=1}^{m} A_i x(k-\tau_i),\\ x(k) &= \phi(k),\quad k \in [-\bar{\tau}, 0],\end{aligned} \tag{7}$$

where $x(k) \in R^n$ is the discrete-time system state, $\tau_i > 0$ is the delay step number of the system, $\phi(k)$ is the initial condition, A, A_i are real constant matrices with appropriate dimensions, and $\bar{\tau}$ is upper bound of τ_i.

Main purpose is to develop a new method to obtain the MADB guaranteeing stability of the discrete-time NCS. In obtaining the results of this paper, the following upper bound for the inner product of two vectors plays an important role:

$$-2a^{\mathrm{T}} Ib \leqslant \inf_{X,Y,Z} \begin{bmatrix} X & Y-I\\ Y^{\mathrm{T}}-I & Z\end{bmatrix}, \tag{8}$$

where $\begin{bmatrix} X & Y\\ Y^{\mathrm{T}} & Z\end{bmatrix} \geqslant 0$ and I denotes an identity matrix with an appropriate dimension. Extending the idea of Eq. (8), the following lemmas are derived.

Lemma 1 (Moon, Park, Kwon, and Lee, 2001). *Assume that $a(\cdot) \in R^{n_a}$, $b(\cdot) \in R^{n_b}$, and $\mathcal{N}(\cdot) \in R^{n_a \times n_b}$ are defined on the interval Ω. Then, for any matrices $X \in R^{n_a \times n_b}$, $Y \in R^{n_a \times n_b}$ and $Z \in R^{n_a \times n_b}$, the following holds:*

$$-2\sum_j a^{\mathrm{T}}(j)\mathcal{N} b(j) \leqslant \sum_j \begin{bmatrix} a(j)\\ b(j)\end{bmatrix}^{\mathrm{T}} \begin{bmatrix} X & Y-\mathcal{N}\\ Y^{\mathrm{T}}-\mathcal{N}^{\mathrm{T}} & Z\end{bmatrix} \begin{bmatrix} a(j)\\ b(j)\end{bmatrix}, \tag{9}$$

where

$$\begin{bmatrix} X & Y\\ Y^{\mathrm{T}} & Z\end{bmatrix} \geqslant 0.$$

Lemma 2. *Let D, E, and Δ be real matrices of appropriate dimensions with $\Delta = \mathrm{diag}\{\Delta_1,\ldots,\Delta_r\}$, $\Delta_i^{\mathrm{T}}\Delta_i \leqslant I_{n_i}$, $i=1,\ldots,r$. Then, for any real matrix $\Lambda = \mathrm{diag}\{\lambda_1 I,\ldots,\lambda_r I\} > 0$, the following inequalities will be true*:

$$D\Delta E + E^{\mathrm{T}}\Delta^{\mathrm{T}}D^{\mathrm{T}} \leqslant D\Lambda D^{\mathrm{T}} + E^{\mathrm{T}}\Lambda^{-1}E, \tag{10}$$

$$D\Delta E + E^{\mathrm{T}}\Delta^{\mathrm{T}}D^{\mathrm{T}} \leqslant D\Lambda^{-1}D^{\mathrm{T}} + E^{\mathrm{T}}\Lambda E. \tag{11}$$

Let us consider a discrete-time NCS equation (7). Theorem 1 presents a delay-dependent stability condition.

Theorem 1. *If there exist P, Q_i, X_I, Y_i and Z_i, $i=1,\ldots,m$ such that*

$$\begin{bmatrix} (1,1) & (1,2)\\ (1,2)^{\mathrm{T}} & (2,2)\end{bmatrix} < 0, \tag{12}$$

$$\begin{bmatrix} X_i & Y_i\\ Y_i^{\mathrm{T}} & Z_i\end{bmatrix} \geqslant 0, \tag{13}$$

where

$$(1,1) \triangleq \begin{bmatrix} -P + \sum_{i=1}^{m}(\bar{\tau}X_i + Y_i + Y_i^{\mathrm{T}} + Q_i) & -Y_M\\ -Y_M^{\mathrm{T}} & -Q_M\end{bmatrix},$$

$$Y_M \triangleq [Y_1 \cdots Y_m],$$

$$Q_M \triangleq \mathrm{diag}\{Q_1,\ldots,Q_m\},$$

$$(1,2) \triangleq \begin{bmatrix} A^{\mathrm{T}}P & \bar{\tau}\sum_{i=1}^{m}(A-I)^{\mathrm{T}}Z_i\\ A_M^{\mathrm{T}}P & \bar{\tau}A_Z^{\mathrm{T}}\end{bmatrix},$$

$$A_M \triangleq [A_1 \cdots A_m],$$

$$A_Z \triangleq [Z_1 A_1 \cdots Z_m A_m],$$

$$(2,2) \triangleq \mathrm{diag}\{-P, -\bar{\tau}Z_1, -\bar{\tau}Z_2, \ldots, -\bar{\tau}Z_m\}$$

then the discrete-time system equation (7) *is asymptotically stable for any time-delay τ_i satisfying $0 \leqslant \tau_i \leqslant \bar{\tau}$, $i=1,\ldots,m$.*

The MADB can be obtained efficiently using the MATLAB LMI Toolbox. System (6) can be represented by Eq. (7) with $N=3$ and $\bar{\tau}$ can be interpreted as $\max\{\bar{\tau}_1, \bar{\tau}_2, \bar{\tau}_3\}$.

The MADB in a discrete-time system is a maximum sampling time which is obtained from Theorem 1. The delay bound of each control loop is used as a parameter in the sampling period and allocation of bandwidth.

3. Event-based scheduling algorithm

This section describes an event-based network scheduling method using the MADB. The modified EDF algorithm is proposed to guarantee real-time transmission of sporadic emergency data. The modified EDF algorithm is that the sporadic data have a high priority than real-time periodic data. In general, a real-time periodic data has a priority higher than non-real-time data. Hence, real-time periodic data is scheduled by EDF algorithm using MADB of each loop as a deadline. After the end of transmission of real-time periodic data, non-real-time message data is transmitted. However, if the sporadic emergency data is occurred, it has a priority over other data. Hence, first of all, a

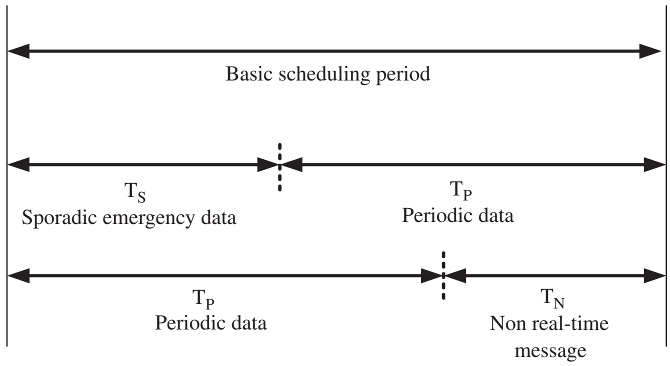

Fig. 3. Configuration of phase in a scheduling period.

sporadic data is scheduled and transmitted. After the transmission of sporadic emergency data, real-time periodic data is transmitted. A basic scheduling period consists of T_S, T_P, and T_N as shown in Fig. 3.

The following assumptions are used in this paper:

1. Sampling time of sensors in a loop is synchronized at starting instant of basic scheduling periods.
2. Control actions of one control loop do not affect other control loops.
3. In networks, communications are error-free. That is, there are no failures in transferring messages.
4. Packets transferred from sensors to controllers of controllers to actuators have the same length.
5. Controller computational delay can be absorbed into either τ_{ca} or τ_{sc} (Walsh & Hong, 2001).

Above assumptions are used to simplify the scheduling condition. The fifth assumption was used for absorbing controller delay time to node data transmission time without loss of generality.

A channel utilization equation (14) is used to determine a sampling period and schedulability:

$$U = \sum_{j=1}^{n} \frac{C_j}{\min(t_j, D_j)} \leqslant 1, \tag{14}$$

where t_j, D_j, and C_j represent period, deadline, and processing time of each task, respectively.

Eq. (14) becomes the channel utilization equation of periodic data:

$$U_P = \sum_{j=1}^{n} \frac{T_P^j}{\min(T^j, T_D^j)} \leqslant 1 - \max(U_S, U_N),$$

$$T_P^j = \sum_{i=1}^{N_S^j} (T_{S_i}^j + T_{O_P}) + \sum_{i=1}^{N_A^j} (T_{C_i}^j + T_{O_P}). \tag{15}$$

U_P, U_S, and U_N are the channel utilization of periodic data, sporadic emergency data, and non-real-time message, respectively. T_β^j is the data transmission time of β in the jth loop (hereinafter, β can be P (periodic data), S (sporadic emergency data), and N (message)). N_A^j and N_S^j represent the number of actuator and sensor in the jth loop. T^j is the sampling period in the jth loop. T_D^j is the MADB in the jth loop. T_{O_β} is the overhead time to transfer β data, $T_{\alpha_i}^j$ is the data transmission time of periodic data in the ith α node in the jth loop (hereinafter, α can be C (controller), A (actuator), and S (sensor)).

Note that the NCS cannot be scheduled if Eq. (15) is not satisfied. In this case, high-speed network protocols should be selected or the number of nodes should be reduced.

The sampling period decision method considers that identical channel utilization for each loop. Considering a channel utilization of one loop, it can be written as

$$\frac{T_P^j}{T^j} = \frac{1 - \max(U_S, U_N)}{n}, \tag{16}$$

where $T^j \leqslant T_D^j$. Eq. (16) becomes

$$T^j = \left\lceil \frac{n}{1 - \max(U_S, U_N)} \right\rceil \times T_P^j, \tag{17}$$

where $\lceil Z \rceil$ is the smallest integer larger than or equal to the value Z. The obtained sampling period of each loop is used as a parameter in the determination of scheduling algorithm. Fig. 4 shows the flow chart of sampling period decision algorithm.

The sporadic data are allocated prior to periodic data, regardless of the deadline. Fig. 5 shows the result of a modified EDF algorithm, where, T_D^1 and T_D^2 are deadline of each loop. Here, T_1 is scheduling period.

The scheduling period is calculated by greatest common divisor (GCD) of sampling periods. The GCD can obtain by $\lceil n/1 - \max(U_S, U_N) \rceil$ of Eq. (17).

4. Simulation

For the simulation, a plant with three DC motors is considered. Each motor has an armature position controller with two sensors and one actuator, which are linked via the network. If the armature inductance (L_a) and viscous frictional coefficient (B_m) are

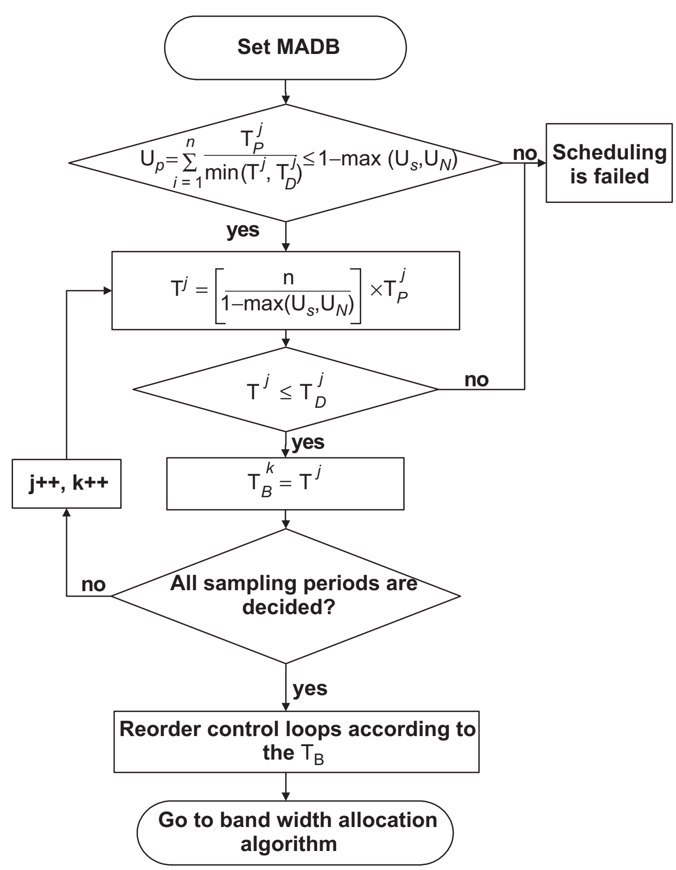

Fig. 4. Flow chart of sampling period decision algorithm.

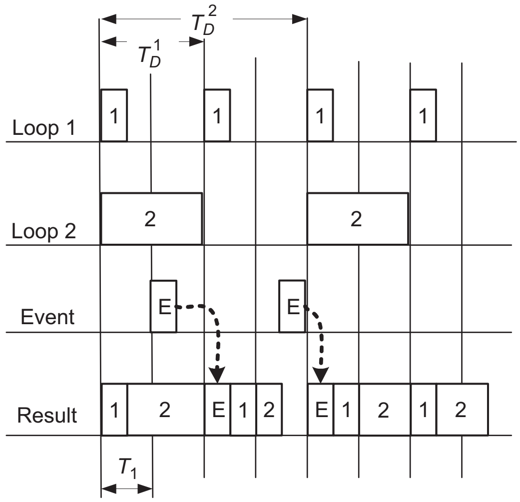

Fig. 5. Example of scheduling result.

negligible, the motor dynamics becomes (Park et al., 2002)

$$
\begin{aligned}
x_p(k+1) &= F_p x_p(k) + G_p u_p(k) \\
&= \begin{bmatrix} -K_i K_b / R_a J & 0 \\ 1 & 0 \end{bmatrix} x_p(k) + \begin{bmatrix} K_i / R_a J \\ 0 \end{bmatrix} u_p(k),
\end{aligned}
$$

$$y_p(k) = x_p(k), \tag{18}$$

where $x_p(k) = [\omega \;\; \theta]^{\mathrm{T}}$, $u_p(k)$ is applied voltage (V), and ω and θ are, respectively, the rotor angular velocity (rad/s) and displacement (rad). R_a, K_i, K_b, and J represent, respectively, the armature resistance, torque constant, back emf constant, and inertia of rotor and load. If a constant gain (K) is used as a state feedback controller, the system equation (6) is changed to

$$x_p(k+1) = F_p x_p(k) + G_p * K x_p(k-\tau),$$

as a single control loop in the NCS, where $\tau = \bar{\tau}_c + \bar{\tau}_{sc} + \bar{\tau}_{ca}$.

For the simulations, the motor in each loop has nominal values such that R_a (Ω), $K_i = 10$ (oz-in/A), $K_b = 0.075$ (V/rad/s), and $J = 0.006$ (oz-in-s^2). The tested motors in each loop have the same nominal values as the previous one except R_a. Motors have the value of $R_a = 14$ (Ω), $R_a = 19$ (Ω), $R_a = 21$ (Ω), respectively. Using Lemma 1, the MADBs are calculated as 3.1, 6.1, and 7.5 ms. Using Theorem 1, the MADBs are calculated as 1.799×10^3, 1.785×10^3, and 1.802×10^3 ms. Table 1 shows the simulation results of MADBs.

The MADBs, obtained using Theorem 1, provide much less conservative delay bounds. Therefore, the number of node and loop can be increased for the scheduling method of the large-scaled NCSs.

From now, the MADBs of each loop can be set as 3, 6, and 7 ms for convenience of calculations. It is assumed that data for the sensor and actuator have 4 bytes. Using the equations in Section 3, the following parameters are given:

$$
\begin{aligned}
&T_D^1 = 3\,\text{ms}, \quad T_D^2 = 6\,\text{ms}, \quad T_D^3 = 7\,\text{ms}, \\
&U_S = 0.17, \\
&U_N^m = 0.16, \\
&N_C^j = N_A^j = 1 \quad \text{for } j = 1,2,3, \\
&N_S^j = 2 \quad \text{for } j = 1,2,3, \\
&N^j = 3 \quad \text{for } j = 1,2,3, \\
&N^* = 1, \quad P = 3,
\end{aligned}
$$

where N^* is the number of sporadic emergency data nodes which do not participate in control loops of NCS.

Table 1
Simulation results of MADBs (ms).

Control loop	R_a (Ω)	Lemma 1 (Park et al., 2002)	Theorem 1
Loop 1	14	3.1	1.77×10^3
Loop 2	19	6.1	1.78×10^3
Loop 3	21	7.5	1.79×10^3

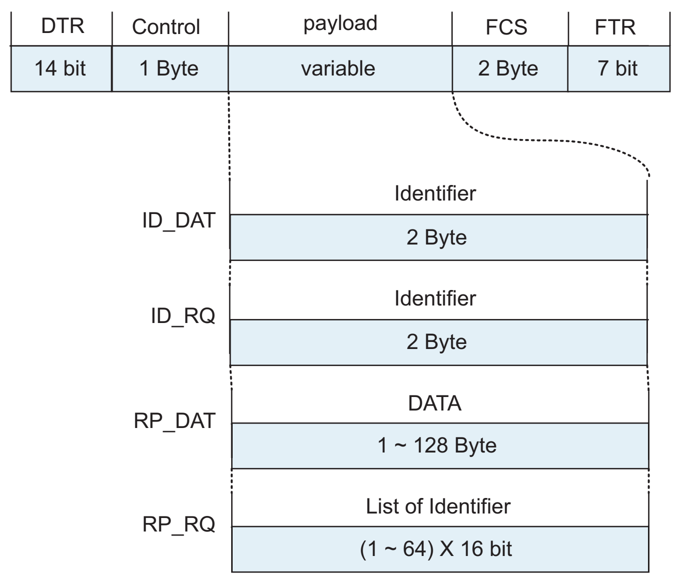

Fig. 6. Frame format of FIP.

The transmission speeds are different according to the given network protocols, but in this example the transmission speed is assumed to be 1 Mbps for convenience of calculations. The data length of sensors and controllers is assumed to be 4 bytes and that of sporadic data is assumed to be 2 bytes. For simplicity of an analysis, it is assumed that buffering and packetizing delays are neglected.

Now consider the application of the scheduling method of the polling control network such as field instrumentation protocol (FIP) (Lorenz & Mammeri, 1995). In FIP, considerable overhead T_{O_S} is required for sporadic data transmission by request frame and its corresponding frame from bus arbitrator. Fig. 6 shows the frame format of FIP. In periodic data transmission, ID_DAT is request frame, transmitted from bus arbitrator, and RP_DAT is response frame. In case of sporadic data transmission, ID_RQ and RP_RQ are added to the periodic one.

Parameters can be given as follows:

$$T_{S_i}^j = T_{C_1}^j = M = 4\ \text{bytes} \times 8\,\mu\text{s/bit} = 32\,\mu\text{s}$$

for $i = 1,2$, $j = 1,2,3$,

$$T_S^M = 2\ \text{bytes} \times 8\,\mu\text{s/bit} = 16\,\mu\text{s}.$$

Message overhead for periodic and sporadic data:

$$T_{O_S}^M = T_{O_P}^M = 45\ \text{bits}\ (RP\_DAT) \times 1\,\mu\text{s/bit} = 45\,\mu\text{s}.$$

Protocol overhead for periodic data:

$$T_{O_P}^P = 61\ \text{bits}\ (ID\_DAT) \times 1\,\mu\text{s/bit} = 61\,\mu\text{s}.$$

Protocol overhead for sporadic data is calculated as

$$
\begin{aligned}
T_{O_S}^P &= \{61(ID\_RQ) + (45+16)(RP\_RQ) + 61(ID\_DAT)\}\ \text{bits} \times 1\,\mu\text{s/bit} \\
&= 183\,\mu\text{s}.
\end{aligned}
$$

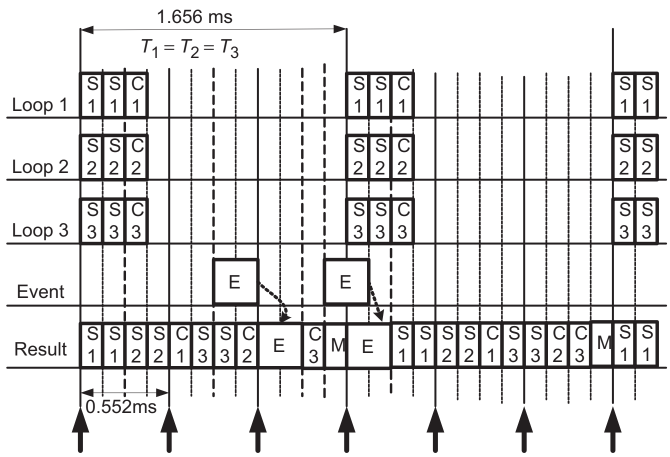

Fig. 7. Scheduling results: 3, 6, 7 ms MADBs.

In the simulation, one time slot, which is transmission time for one packet of real-time periodic data, can be calculated as $M + T^M_{O_P} + T^P_{O_P} = 138\,\mu s$. In Table 1, MADBs can be determined 1.799, 1.785, and 1.780 s by Theorem 1. Hence, the sampling period can be reduced to 1.6 ms and the scheduling period is 0.552 ms. Since the number of transmission node in each loop is identical, sampling periods of each loop have the same period. The scheduling results are shown in Fig. 7.

5. Conclusion

In this paper, the MADBs are obtained for the stability of the discrete-NCS using LMI formulation, and are used as the basic parameter for an event-based scheduling method for the NCS. The scheduling method for the NCS can schedule efficiently to guarantee real-time transmission for sporadic data. The presented sampling period decision algorithm is useful, as it provides a fair channel utilization to loops, and calculates sampling period easily. The modified EDF algorithm schedules sporadic data prior to periodic data. Hence, real-time performance of sporadic data is improved.

As a future direction of this work, one may be consider non-real-time message with real-time data and more realistic bounds on NCSs including packet retransmission.

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Appendix A

Proof (*Lemma 2*). From the fact $(\Lambda^{1/2}D^T - \Lambda^{-1/2}\Delta E)^T(\Lambda^{1/2}D^T - \Lambda^{-1/2}\Delta E) \geqslant 0$, it follows that $D\Delta E + E^T\Delta^T D^T \leqslant D\Lambda D^T + E^T\Delta^T\Lambda^{-1}\Delta E$. For Δ and Λ, we have the relation

$$\Delta^T\lambda^{-1}\Delta = \mathrm{diag}\{\lambda_1^{-1}\Delta_1^T\Delta_1, \ldots, \lambda_r^{-1}\Delta_r^T\Delta_r\} \leqslant \mathrm{diag}\{\lambda_1^{-1}I_{n_1}, \ldots, \lambda_r^{-1}I_{n_r}\} = \lambda^{-1}.$$

Hence, $E^T\Delta^T\Lambda^{-1}\Delta E \leqslant E^T\Lambda^{-1}E$. From this, Eq. (10) follows. The proof of Eq. (11) starts from the fact $(\Lambda^{-1/2}D^T - \Lambda^{1/2}\Delta E)^T(\Lambda^{-1/2}D^T - \Lambda^{1/2}\Delta E) \geqslant 0$. The remaining procedure is quite clear, hence omitted. This completes the proof. □

Appendix B

Proof (*Theorem 1*). Choose a Lyapunov functional as follows:

$$V(k) \triangleq V_1(k) + V_2(k) + V_3(k), \tag{19}$$

where

$$V_1(k) \triangleq x(k)^T P x(k),$$
$$V_2(k) \triangleq \sum_{i=1}^{m} \sum_{\beta=-\tau_i}^{-1} \sum_{j=k+\beta+1}^{k} \delta x(j)^T Z \delta x(j),$$
$$\delta x(j) \triangleq [x(j) - x(j-1)],$$
$$V_3(k) \triangleq \sum_{i=1}^{m} \sum_{j=k-\tau_i}^{k-1} x(j)^T Q_i x(j).$$

Since it holds that

$$x(k-h_i) = x(k) - \sum_{j=k-\tau_i+1}^{k} [x(j) - x(j-1)]$$

the discrete-time system (7) can be written as

$$x(k+1) = \left(A + \sum_{i=1}^{m} A_i\right)x(k) - \sum_{i=1}^{m}\left\{A_i \sum_{j=k-\tau_i+1}^{k}[x(j) - x(j-1)]\right\} \tag{20}$$

and thus increment of V_1 satisfies the relation

$$\Delta V_1(k) = V_1(k+1) - V_1(k) = x(k)^T\left(A + \sum_{i=1}^{m}A_i\right)^T P\left(A + \sum_{i=1}^{m}A_i\right)x(k) - 2x(k)^T\left(A + \sum_{i=1}^{m}A_i\right)^T P\Gamma + \sum_{i=1}^{m}(\delta x_{h_i}(k)^T A_i^T P A_i \delta x_{h_i}(k)) - x(k)^T P x(k),$$

$$\Gamma = \sum_{i=1}^{m}\left\{A_i \sum_{j=k-\tau_i+1}^{k}[x(j) - x(j-1)]\right\},$$
$$\delta x_{h_i}(k) = [x(k) - x(k-h_i)]. \tag{21}$$

Defining $a(\cdot)$, $b(\cdot)$, and $\mathcal{N}$ in Eq. (9) as $a(j) \triangleq x(k)$, $b(j) \triangleq x(j) - x(j-1)$ and $\mathcal{N} \triangleq (A + \sum_{i=1}^{m}A_i)^T P A_i$ for all $i = 1, 2, \ldots, m$ and $j \in [k-h+1, k]$, and applying Lemma 1

$$\Delta V_1(k) \leqslant x(k)^T H_1 x(k) + \sum_{i=1}^{m} 2x(k)^T[-Y_i + A^T P A_i]x(k-h_i) + \sum_{i=1}^{m} x(k-h_i)^T A_i^T P A_i x(k-h_i) + \sum_{i=1}^{m}\sum_{j=k-h_i+1}^{k} \delta x(j)^T Z_i \delta x(j),$$

$$H_i \triangleq A^T P A - P + \sum_{i=1}^{m}(\bar{h}X_i + Y_i + Y_i^T). \tag{22}$$

Since $\Delta V_2(k)$ and $\Delta V_3(k)$ yield the relation

$$\Delta V_2(k) = \sum_{i=1}^{m} h_i H_2^T Z_i H_2 - \sum_{i=1}^{m}\sum_{j=k-h_i+1}^{k} \delta x(j)^T Z_i \delta x(j), \tag{23}$$

$$H_2 = (A - I)x(k) + A_1 x(k-h),$$

$$\Delta V_3(k) = \sum_{i=1}^{m}[x(k)^{\mathrm{T}}Q_i x(k) - x(k-h_i)^{\mathrm{T}}Q_i x(k-h_i)]. \tag{24}$$

Therefore,

$$\Delta V(k) = \Delta V_1(k) + \Delta V_2(k) + \Delta V_3(k) \leqslant \begin{bmatrix} x(k) \\ X_h(k) \end{bmatrix} \begin{bmatrix} (1,1)_h & (1,2)_h \\ (1,2)_h^{\mathrm{T}} & (2,2)_h \end{bmatrix} \begin{bmatrix} x(k) \\ X_h(k) \end{bmatrix}, \tag{25}$$

where

$$X_h(k) \triangleq [x(k-h_1)\cdots x(k-h_m)]^{\mathrm{T}},$$

$$(1,1)_h \triangleq A^{\mathrm{T}}PA - P + \sum_{i=1}^{m}(\bar{h}X_i + Y_i Y_i^{\mathrm{T}}) + \sum_{i=1}^{m}(\bar{h}(A-I)^{\mathrm{T}}Z_i(A-I)) + \sum_{i=1}^{m}Q_i,$$

$$(1,2)_h \triangleq [B_1 \cdots B_m],$$

$$B_i \triangleq -Y_i + \bar{h}(A-I)^{\mathrm{T}}Z_i A_i + A^{\mathrm{T}}PA_i,$$

$$(2,2)_h \triangleq \mathrm{diag}\{D_1 \cdots D_m\},$$

$$D_i \triangleq A_i^{\mathrm{T}}PA_i + \bar{h}A_i^{\mathrm{T}}Z_i A_i - Q.$$

Then, using the Lyapunov–Krasovskii stability theorem and Schur complement, the discrete-time system (7) is asymptotically stable if Eqs. (12) and (13) hold. This completes the proof. □

References

Asok, R., & Yoram, H. (1988). Integrated communication and control systems: Part II—design consideration. *Journal of Dynamic Systems, Measurement, and Control*, *110*, 374–381.

Beauvais, J. P., & Deplanche, A.-M. (1995). Heuristics for scheduling periodic complex real-time tasks in a distributed system. In *Proceedings of IECON'95* (pp. 55–60).

Branicky, M. S., Phillips, S. R., & Zhang, W. (2000). Stability of networked control systems: Explicit analysis of delay. In *Proceedings of the American control conference* (Vol. 4, pp. 2352–2357).

Cavalieri, S., Stefano, A., & Mirabella, O. (1995). Pre-run-time scheduling to reduce schedule length in the fieldbus environment. *IEEE Transactions on Software Engineering*, *21*(11), 865–880.

Hong, S. H. (1995). Scheduling algorithm of data sampling times in the integrated communication and control systems. *IEEE Transactions on Control Systems Technology*, *3*(2), 225–231.

Hong, Y., & Walsh, G. C. (2001). Real-time mixed-traffic wireless networks. *IEEE Transactions on Industrial Electronics*, *48*, 883–890.

Kim, D. S., Lee, Y. S., Kwon, W. H., & Park, H. S. (2003). Maximum allowable delay bounds of networked control system. *Control Engineering Practice*, *11*, 1301–1313.

Krtolica, R., Özgüner, Ü., Chan, H., Göktas, H., Winkelman, J., & Liubakka, M. (1994). Stability of linear feedback systems with random communication delays. *International Journal of Control*, *59*(4), 925–953.

Li, X., & de Souza, C. E. (1997a). Criteria for robust stability and stabilization of uncertain linear systems with state delays. *Automatica*, *33*(9), 1657–1662.

Li, X., & de Souza, C. E. (1997b). Delay-dependent robust stability and stabilization of uncertain linear delay systems: A linear matrix inequality approach. *IEEE Transactions on Automatic Control*, *42*(8), 1144–1148.

Lian, F.-L., Moyne, J., & Tilbury, D. (2002). Network design consideration for distributed control systems. *IEEE Transactions on Control Systems Technology*, *10*, 297–307.

Lorenz, P., & Mammeri, Z. (1995). Real-time software architecture: Application to FIP fieldbus. In *Proceedings of AARTC* (pp. 415–423).

Moon, Y. S., Park, P. G., Kwon, W. H., & Lee, Y. S. (2001). Delay-dependent robust stabilization of uncertain state-delayed systems. *International Journal of Control*, *74*, 1447–1455.

Nilson, J. (1998). *Real time control systems with delays*. Ph.D. Dissertation, Department of Automatic control, Lund Institute of Technology, Lund, Sweden.

Nilson, J., Bermhardsson, B., & Wittermark, B. (1998). Stochastic analysis of control of real time systems with random time delays. *Automatica*, *34*, 57–64.

Park, H. S., Kim, Y. H., Kim, D. S., & Kwon, W. H. (2002). A scheduling method for network based control systems. *IEEE Transactions on Control Systems Technology*, *10*, 318–330.

Park, P. G. (1999). A delay-dependent stability criterion for systems with uncertain time-invariant delays. *IEEE Transactions on Automatic Control*, *44*(4), 876–877.

Raja, P., Vijayananda, K., & Decotignie, J. D. (1993). Polling algorithms and their properties for fieldbus networks. In *Proceedings of IECON'93* (pp. 530–534).

Vatanski, N., Georges, J.-P., Aubrun, C., Rondeau, E., & Jamsa-Jounela, S.-L. (2008). Networked control with delay measurement and estimation. *Control Engineering Practice*, in press, doi: 10.1016/j.conengprac.2008.07.004.

Walsh, G. C., & Hong, Y. (2001). Scheduling of networked control systems. *IEEE Control Systems Magazine*, *21*(1), 57–65.

Walsh, G. C., Hong, Y., & Bushnell, L. (1999). Stability analysis of networked control systems. In *Proceedings of the American control conference* (Vol. 4, pp. 2876–2880).

Yoram, H., & Asok, R. (1988). Integrated communication and control systems: Part I—analysis. *Journal of Dynamic Systems, Measurement, and Control*, *110*, 367–373.

Zhang, W., Branicky, M. S., & Phillips, S. M. (2001). Stability of networked control systems. *IEEE Control Systems Magazine*, *21*(1), 84–99.

Zuberi, K. M., & Shin, K. G. (1997). Scheduling messages on controller area network for real-time CIM applications. *IEEE Transactions on Robotics and Automation*, *13*, 310–316.